Bell’s Theorem, The Ultimate Shock of Quantum Mechanics

Bohr: “For those who are not shocked when they first come across quantum theory cannot possibly have understood it.”

“Anyone who is not shocked by quantum theory has not understood a single word.”

Feynman: “I think I can safely say that nobody understands quantum mechanics.”

“Shut up and calculate!”

Mermin on Einstein:
“We often discussed his notions on objective reality. I recall that during one walk Einstein suddenly stopped, turned to me and asked whether I really believed that the moon exists only when I look at it.”

Einstein:
“God does not play dice with the universe”

Probabilistic interpretation for single particle, weird, but still ok. Like Coin Toss? Supplementary conditions (Hidden Variables?)

Entanglement (2 particle wave function, disaster!)

$1/\sqrt{2} \ (|+> \rightarrow \pm \text{ or } - > |+>)$ for electrons -> +, s(z)=1/2, -, s(z)= -1/2

$1/\sqrt{2} \ (|R>|R> + |L>|L>) = 1/\sqrt{2} \ (|x>|x> - |y>|y>)$ for photons

$|R> \rightarrow s(z)=+1, \ |L> \rightarrow s(z)= -1$

$|R> = -1/\sqrt{2} \ (|x> + i |y>) \ |L> = 1/\sqrt{2} \ (|x> - i |y>)$

Measuring one determines the property of the other without disturbing it, no matter how far they are.
EPR (Einstein, Podolsky and Rosen, 1935) on Reality

If without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity), the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity. i.e both $S(z,B)$ and $S(x, B)$ known with certainty by measuring $A$

Locality:
Maximum speed with which two systems can communicate is the speed of light. So two objects at space like distances cannot communicate.

EPR -> Local Realism

Einstein (March 1947):

"I cannot seriously believe in [the quantum theory] because it cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance.

Einstein (March 1948):

"That which really exists in B should ...not depend on what kind of measurement is carried out in part of space A; it should also be independent of whether or not any measurement at all is carried out in space A. If one adheres to this program, one can hardly consider the quantum-theoretical description as a complete representation of the physically real. If one tries to do so in spite of this, one has to assume that the physically real in B suffers a sudden change as a result of a measurement in A. My instinct for physics bristles at this."
EPR-> Quantum Mechanics Incomplete, some supplementary conditions (Hidden variables) 1935. Bohm. Then QED. Tremendous successes of QM continued.

Bell’s Theorem (1964) (Von Neumann’s proof, assumptions too restricted, conclusion wrong)

Bell initially thought hidden variables would win experimentally! In his heart he favored hidden variables!

“No physical theory of local hidden variables (local reality) can ever reproduce all of the predictions of quantum mechanics.”

“Some physicists are convinced that [Bell’s theorem] is the most important single work, perhaps, in the history of physics.”

Henry Stapp, a particle theorist at Berkeley:
“Bell’s theorem is the most profound discovery of science.”

An anonymous physicist’s comment after the APS session on Bell’s theorem:

“Anybody who’s not bothered by Bell’s theorem has to have rocks in his head.”

Simple logical inequality:

\[ N(A, \text{not } B) + N(B, \text{not } C) \geq N(A, \text{not } C) \]

Proof:

\[ N(A, \text{not } B, C) + N(\text{not } A, B, \text{not } C) \geq 0 \]

Add \( N(A, \text{not } B, \text{not } C) + N(A, B, \text{not } C) \) to both sides

\[ N(A, \text{not } B) + N(B, \text{not } C) \geq N(A, \text{not } C) \]
Bell’s Theorem: Q.M. violates this. Ideas of reality and Hidden Variables have to respect this inequality!!

Two particles flying opposite to each other, two detectors Right And Left

Electrons (spin $\frac{1}{2}$) $1/\sqrt{2}$ (|+⟩ |−⟩ $\pm$ |−⟩|+⟩)

A- Rt 0 deg, B –Rt 45 deg, C- Rt 90 deg. All spin-up
Stern-Gerlach set-up
Inhomogeneous magnetic field +1/2, -1/2 deflected in opposite directions –can block one---Stern-Gerlach Filter- ±1

Problem: Heisenberg: 0 deg measurement disturbs 45 deg which follows. Entanglement ->

Rt not up 45 deg =Rt down 45 deg= Lt up 45 deg

Inequality:

$N(\text{Rt 0 deg. up, Lt 45 deg up}) + N(\text{Rt 45 deg. up, Lt 90 deg. up}) \geq N(\text{Rt 0 deg. up, Lt. 90 deg. up})$

Q.M. eigen states of $\sigma$. $n$ (n at angle $\theta$) = $\sigma(z)\cos \theta + \sigma(x) \sin \theta$

$$= \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix}$$
\[(\cos \theta/2, \sin \theta/2), \quad (-\sin \theta/2, \cos \theta/2)\]

\[(1,0) = \cos \theta/2 (\cos \theta/2, \sin \theta/2) - \sin \theta/2 (-\sin \theta/2, \cos \theta/2)\]

\[(0,1) = \sin \theta/2 (\cos \theta/2, \sin \theta/2) + \cos (\theta/2) (-\sin \theta/2, \cos \theta/2)\]

\[p^{++} = p^{--} = 1/2 \sin (\theta/2) **2, \quad p^{+-} = p^{-+} = 1/2 \cos (\theta/2) **2\]

\[N \propto p\]

Inequality:

\[\sin (22.5) **2 + \sin (22.5) **2 \geq \sin(45) **2\]

\[0.292 \geq 0.5 \text{ badly violated!!}\]

**Photons**

\[1/\sqrt{2} (|x> \pm |y> |y>) \text{ for photons } x \rightarrow +, y \rightarrow -\]

Classical E vector ok. Malus' law

\[\theta \text{ (angle between two polarizers)}\]

\[p^{++} = p^{--} = \frac{1}{2} (\cos \theta)**2, \quad p^{+-} = p^{-+} = \frac{1}{2}(\sin (\theta)**2\]

\[A = \text{Rt 0 deg. x,} \quad B = \text{Rt 22.5 deg. x,} \quad C = \text{Rt. 45 deg. x}\]
not B= Rt. 22.5 deg. y, not C= Rt. 45 deg. y

Heisenberg again! Entanglement:

Not B= Lt. 22.5 deg. y, not C= Lt. 45 deg. y
Inequality:
\[ \sin(22.5)^2 + \sin(22.5)^2 \geq \sin(45)^2 \]

0.292 \geq 0.5 badly fails.

Now what? Ordinary logic fails.
Picture of reality—"parameters exist whether they are measured or not" fails. Non-local connectivity wins!

Bell thought hidden variables would win experimentally!

Photons Experiments: Many

Clauser, Horne, Shimony, Holt Inequality (‘70s)
Clauser, Freedman -expt
Aspect et al (‘80s)
Zeilinger’s group
Angles (a,a’)-one side (b,b’)-other side

A,B,A’,B’ results of measurements—± = ± 1
Correlation Function
Theory \( \rightarrow E(a,b) = \sum AB \) p(A,B) = (p++ + p--) – (p- + p+–)
Expt. $\rightarrow E(a,b) = \frac{1}{N} \Sigma A(n)B(n)$, $N$ trials

Def. $X = AB - AB' + A'B + A'B'$

$= A(B-B') + A'(B+B')$

Real spin components (hidden variables), although quantized, $\pm 1$

$-2 \leq <X> \leq 2$

Classically, $A,B$ components of spin along some axis, same Result.

Average $<X> = E(a,b) - E(a,b') + E(a',b) + E(a',b')$

Expt. Measure $<X>$
Q.M. Theory $\rightarrow E(a,b) = \cos 2\theta$

Q.M. $<X> = \cos 45^\circ - \cos 135^\circ + \cos 45^\circ + \cos 45^\circ = 2\sqrt{2}$

Angles varied at random, faster than time required for light signals to go from $a$ to $b$, settings changed after photons left source! Switching $\rightarrow$ acousto-optical modulator or electro-optical modulator.

Aspect, switching time-10 ns. Travel time 43 ns.
Wiehs, Zeilinger, distance 400m, 100ns, 1.3$\mu$s
Later expts-distances several tens of kilometers.
All Expts agree –violate inequality within 40 to 240 s.d.!!
Mermin:
Quantum mechanics violates Bell inequality for a state of $n$ spin-1/2 particles by an amount that grows exponentially with $n$.

Hidden Variables $\lambda$, local $: A$ independent of $b$.

$E(a,b) = \int d\lambda \rho(\lambda) A(\lambda,a) B(\lambda,b)$

Inequality still good for any form of $\lambda$, distribution, infinite dimensional matrix etc.
Greenberger–Horne–Zeilinger states (‘90s)
Three particle correlations- do not require inequalities.

\[ \Sigma(a)=\sigma(a,x)\sigma(b,y)\sigma(c,y); \Sigma(b)=\sigma(a,y)\sigma(b,x)\sigma(c,y) \]
\[ \Sigma(c)=\sigma(a,y)\sigma(b,y)\sigma(c,x) \]
\[ \Sigma=\sigma(a,x)\sigma(b,x)\sigma(c,x) = -\Sigma(a)\Sigma(b)\Sigma(c) \]

\( \sigma(x),\sigma(y) \) anticommute for same particle, commute for different particles.
\[ \sigma(x)|+,->=|-++,\sigma(y)|+,->= \pm|-,++> \]

\( \Sigma(a), \Sigma(b), \Sigma(c), \Sigma \) commute.
\[ \Psi=\frac{1}{\sqrt{2}} (|+,+,+-|-,+->) \] common eigenvector, eigenvalue +1 for three, -1 for fourth.

Two particles same value, third one must be +1, if two differ, third must be -1. Prediction without disturbing the third one
-> Element of reality!
Can choose x,y during flight!
With hidden variables
\[ A(x)B(y)C(y)A(y)B(x)C(y)A(y)B(y)C(x) = A(x)B(x)C(x) \]
Contradiction

Do not need large statistics. Few measurements reveal contradiction, Agreement with Q.M. Farewell to local realism!!
For whom the Bell tolls?

Expts -> Zeilinger et al., Pan et al, Kwiat et al.
Many expts for entanglement in position, mometa etc, measurement of interference effects -> similar results.

If time permits delayed choice and quantum eraser -> Scully’s expt
Figure 10 - Two-photon selective excitation of the 4p$^2$ $^1S_0$ state of Calcium with a Krypton ion laser and a tunable dye laser. From this state, the atom radiative decay can only deliver the pair of entangled photons ($\nu_1, \nu_2$).

Figure 15 - Timing-experiment with optical switches ($C_1$ and $C_2$). The switch $C_1$ followed by the two polarizers in orientations $a$ and $a'$ is equivalent to a single polarizer switched between the orientations $a$ and $a'$. A switching occurs approximatively each 10 ns. A similar setup, independently driven, is implemented on the second side. In our experiment, the distance $L$ between the switches was large enough (13 m) that the time of travel of a signal between the switches at the velocity of light (43 ns) was significantly larger than the delay between two switchings (about 10 ns) and the delay between the emission between the two photons (5 ns average).
Fig. 6.9. Configuration of a GHZ type of experiment.

Figure 1 Experimental set-up for Greenberger–Horne–Zeilinger (GHZ) tests of quantum nonlocality. Pairs of polarization-entangled photons are generated by a short pulse of ultraviolet light (~200 fs, \(\lambda = 394\) nm). Observation of the desired GHZ correlations requires fourfold coincidence and therefore two pairs. The photon registered at T is always \(H\) and thus its partner in \(a\) must be \(V\). The photon reflected at the polarizing beam-splitter (PBS) at arm \(a\) is always \(V\), being turned into equal superposition of \(V\) and \(H\) by the \(\lambda/2\) plate, and its partner in arm \(b\) must be \(H\). Thus if all four detectors register at the same time, the two photons in \(D_1\) and \(D_2\) must either both have been \(VV\) and reflected by the last PBS or \(HH\) and transmitted. The photon at \(D_2\) was therefore \(H\) or \(V\), respectively. Both possibilities are made indistinguishable by having equal path lengths via \(a\) and \(b\) to \(D_1\) and \(D_2\) and by using narrow bandwidth filters (\(F = 4\) nm) to stretch the coherence time to about 500 fs, substantially longer than the pulse length. This effectively erases the prior correlation information and, owing to indistinguishability, the three photons registered at \(D_1\), \(D_2\) and \(D_3\) exhibit the desired GHZ correlations predicted by the state of equation (1), where for simplicity we assume the polarizations at \(D_3\) to be defined at right angles relative to the others. Polarizers oriented at 45° and \(\lambda/4\) plates in front of the detectors allow measurement of linear \(H/V\) (circular \(RV\) ) polarization.
How a Quantum Eraser Works

How quantum particles behave can depend on what information about them can possibly be accessed. A quantum eraser eliminates some information and thereby restores the phenomenon of interference. The eraser’s action is most easily understood by considering a “double-slit” experiment [below].

Particles sent through two slits generate bands (called fringes) on a detector screen when large numbers arrive at some regions (blue) and very few arrive at other regions (white). This interference pattern arises only if each particle could have traveled through both slits to arrive at the screen (arrows).

The fringes do not appear if the particles interact with something that could thereby be used to ascertain each particle’s location at the slits. For example, a photon of light (yellow line) might scatter from the particle and reveal that it went through the right-hand slit. The photon need not be detected—all that matters is that the “which slit?” information in principle could be determined if it were to be detected.

A quantum eraser erases the “which slit?” information. If the particle scatters a photon, a lens could make it impossible to ascertain which slit the photon came from. In that case, the corresponding particle apparently goes through both slits, as before, and fringes can be observed. The strangest feature of this quantum erasing is that the behavior of the particle at the slits seemingly depends on what the photon encounters after the particle has passed through the slit(s).
Fig. 2 from paper, with emphasis on segments of the experimental procedure during which no which-path information is available in principle.

Fig. 3 — Joint Detection $R_{01}$

Fig. 4 — Joint Detection $R_{02}$
IDEAL QUANTUM TELEPORTATION relies on Alice, the sender, and Bob, the receiver, sharing a pair of entangled particles A and B (green). Alice has a particle that is in an unknown quantum state \( X \) (blue). Alice performs a Bell-state measurement on particles A and X, producing one of four possible outcomes. She tells Bob about the result by ordinary means. Depending on Alice's result, Bob leaves his particle unaltered (1) or rotates it (2, 3, 4). Either way, it ends up a perfect replica of the original particle X.