Casimir Pressure Between Half-Spaces in a Fully Non-Equilibrium Situation

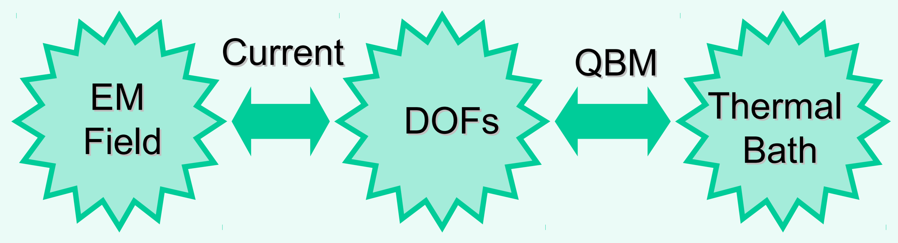
Adrián E. Rubio López[1]*; Gustavo J. Turiaci[2]**; Fernando C. Lombardo[1]; Francisco D. Mazzitelli[3].
[1] Departamento de Física, FCEyN, UBA; [2] Physics Department, Princeton University, Princeton; [3] Centro Atómico Bariloche (CNEA).
*arubio@df.uba.ar; **turiaci@princeton.edu.

In this work we present a detailed study and results on ***non-equilibrium Casimir pressure*** produced by the electromagnetic field (EM) between two half-spaces, modeled as dissipative media by a continuous distribution of quantum degrees of freedom (DOFs). These DOFs are quantum harmonic oscillators representing the polarization vectors of the half-spaces, and they present a double coupling, one Brownian-type coupling to a thermal bath (also composed by sets of quantum harmonic oscillators) and other current-type coupling to the EM-field. Each part of the composite system (field, DOFs and baths) are described at the beginning by thermal states with different temperature (***thermodynamic non-equilibrium***). Using the in-in (CTP) formalism due to Schwinger and Keldysh, we compute the coarse-graining effective action, tracing out the DOFs and the bath. It is shown that the Casimir pressure presents different contributions associated to both transient and steady situations (analysing the ***full non-equilibrium time evolution***). In the same way, from the thermodynamical point of view, the pressure also can be split in different thermal contributions associated to each part of the system. Finally we study the fully non-equilibrium dynamics of heat transfer between the half-spaces via the Poynting vector. Our results are useful to generalize previous non-equilibrium results valid in the long-time limit (steady situation).

The Model: Field, DOFs and Baths

In recent years it has gained great importance the need of including ***fully non-equilibrium*** effects in the calculations of the ***Casimir effect*** [1,2]. The CTP or Schwinger-Keldysh formalism provides the natural theoretical approach for this situation allowing the study of its time evolution. The model consists of a system composed of three parts: field, DOFs of the wall and thermal baths; which begin to interact at $t = t_0$. The action associated to this composite system is given by:

$$S[\mathbf{E},\mathbf{P},\mathbf{Y}] \equiv S_{\text{EM}}[A_\mu] + S_{\text{D}}[\mathbf{P}] + S_{\text{EM-P}}[A_\mu,\mathbf{P}] + S_{\text{R}}[\mathbf{Y}] + S_{\text{R-P}}[\mathbf{Y},\mathbf{P}]$$
$$= \int dt d^3x \left\{ \frac{\epsilon_0}{2}\mathbf{E}^2 - \frac{1}{2\mu_0}\mathbf{B}^2 + \frac{g(\mathrm{x})}{2\epsilon_0\omega_0^2\beta}\left(\dot{\mathbf{P}}^2 - \omega_0^2\mathbf{P}^2\right) - g(\mathrm{x})\mathbf{E}\cdot\mathbf{P} + \int_0^\infty d\nu g(\mathrm{x})\left(\frac{\rho}{2}\dot{\mathbf{Y}}_\nu^2 - \frac{\rho\nu}{2}\mathbf{Y}_\nu^2 - f(\nu)\mathbf{P}\cdot\mathbf{Y}_\nu\right)\right\}$$



Initially, the whole system is assumed to be uncorrelated and each part is assumed as in thermal equilibrium.

In- in (CTP) Effective Action

Our aim now is to calculate the effective action with the in - in formalism by means of closed time path (CTP) integral:

$$e^{i\left(S_{EM}[A^a_\mu]+S_{IF}[A^a_\mu]\right)} = \int_{CTP} \mathfrak{D}\mathbf{P}\mathfrak{D}\mathbf{Y}_\nu \; e^{iS[A^a_\mu,\mathbf{P}^a,\mathbf{Y}^a_\nu]}$$

where the influence action is given by:

$$S_{IF} = \int d^4x \int d^4x \left\{\Delta E_i(x)D(x,x')\Sigma E_i(x') + \frac{i}{4}\Delta E_i(x)N_{ij}(x,x')\Delta E_j(x')\right\}$$

The kernels are the ones found in Ref. [2].
We can re-write this CTP-influence action including an ***stochastic force*** through Feynman's trick, obtaining:

$$S_{IF} = \int d^4x \int d^4x \Delta E_i(x)D(x,x')\Sigma E_i(x') + \int d^4x \xi_i(x)\Delta E_i(x)$$

where the stochastic force is characterized by a gaussian distribution satisfying:

$$\langle\{\xi_j(x),\xi_k(x')\}\rangle_\xi = N_{jk}(x,x')$$

Under this conditions, the correlation for symmetrized two-point function is given by (Ref. [3]):

$$\langle A_1 A_2\rangle^{Total} = \langle A_{\xi,1}A_{\xi,2}\rangle_{\text{i.c.},\xi}$$

Maxwell Equations and Pressure

From the set of modified Maxwell equations:

$$\nabla\times\mathbf{B} - \frac{\partial}{\partial t}\mathbf{E} - \frac{\partial}{\partial t}\int_{t_0}^t dt' D(t,t';\mathrm{x})\mathbf{E}(t',\mathrm{x}) = \frac{\partial}{\partial t}\xi(t,\mathrm{x})$$
$$\nabla\times\mathbf{E} + \frac{\partial}{\partial t}\mathbf{B} = 0$$

the field correlation is written as:

$$\langle\mathbf{E}(s_1,\mathrm{x}_1)\,[\mathbf{E}(s_2,\mathrm{x}_2)]^\dagger\rangle = \int d^3x' \int d^3x'' \left\langle \left[\overline{\mathbf{G}}(s_1,\mathrm{x}_1,\mathrm{x}')\cdot\mathbf{P}(s_1,\mathrm{x}')\right]\left[\overline{\mathbf{G}}(s_2,\mathrm{x}_2,\mathrm{x}'')\cdot\mathbf{P}(s_2,\mathrm{x}'')\right]^\dagger\right\rangle_{\text{i.c.},\xi}$$

where G is the electric field ***Green function*** and P stands for its source related with the stochastic force and the field initial conditions.
The pressure in the vacuum gap then reads:

$$P^{Total}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l) = P^{neq}_{(s,s'^*)}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l) + 2P_{(s)}(T_{EM},l)$$

where each term is given by:

$$P^{neq}_{(s,s'^*)}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l) = -\frac{1}{8\pi}\int_{a-i\infty}^{a+i\infty}\frac{ds}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{ds'}{2\pi i}\; e^{(s+s'^*)t}\left[\Theta_{\delta\nu}\langle E_\delta(s,\mathrm{x})E_\nu(s'^*,\mathrm{x}')\rangle\right]\Big|_{\mathrm{x}=\mathrm{x}'}$$
$$P_{(s)}(T_{EM},l) = \frac{1}{8\pi}\int_{a-i\infty}^{a+i\infty}\frac{ds}{2\pi i}\,\frac{e^{st}}{s}\,\Lambda_{\alpha\beta}\,\epsilon_{\alpha\gamma\delta}\left[\partial_\gamma\langle E_\delta(s,\mathrm{x})B_\beta(t_0,\mathrm{x}')\rangle\right]\Big|_{\mathrm{x}=\mathrm{x}'}$$

with:

$$\Theta_{\delta\nu} = \Lambda_{\alpha\beta}\left(\delta_{\alpha\delta}\delta_{\beta\nu} + \frac{1}{ss'^*}\epsilon_{\alpha\gamma\delta}\epsilon_{\beta\eta\nu}\partial_\gamma\partial'_\eta\right)$$

The Pressure Contributions

Since the stochastic force has null expectation value, the correlation of the source splits in two contributions, one associated to the stochastic force and other one associated to the field initial conditions:

$$\langle P_k(s_1,\mathrm{x}')P_l(s_2^*,\mathrm{x}'')\rangle = \langle P_k(s_1,\mathrm{x}')P_l(s_2^*,\mathrm{x}'')\rangle_\xi + \langle P_k(s_1,\mathrm{x}')P_l(s_2^*,\mathrm{x}'')\rangle_{i.c.}$$

Thus, this property also holds for the electric field correlation.

For the case of the stochastic contribution, it can be also proved that the pressure separates in distinct contributions of the DOFs and baths of each half-space (***thermal non-equilibrium***):

$$P^{neq}_{s,s'^*}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l) = \sum_{m=1}^{2}\left(P^{(X)}_{s,s'^*}(T^{(m)}_X,l) + P^{(P)}_{s,s'^*}(T^{(m)}_P,l)\right) + P_{s,s'^*}(T_{EM},l)$$

From a dynamical point of view, it can also be proved that the steady situation in Ref. [1] can be re-obtained as a long-time contribution (***non-equilibrium time evolution***):

$$P^{neq}_{s,s'^*}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l) = P^\infty_s(T^{(1)}_X,T^{(2)}_X,l) + P^{Trans}_{s,s'^*}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l)$$

The other term in the pressure is also transient and depends on the field's initial temperature.

Heat Transfer

The heat transfer can be studied by the (time-dependent) ***Poynting vector*** evolution, which is given by:

$$S_i^{Total}(t,\mathrm{x}) = \frac{1}{4\pi}\int_{a-i\infty}^{a+i\infty}\frac{ds}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{ds'}{2\pi i}e^{(s+s'^*)t}\epsilon_{ijk}\langle E_j(s,\mathrm{x})B_k(s'^*,\mathrm{x}')\rangle\Big|_{\mathrm{x}'=\mathrm{x}}$$

As in the pressure case, it can be proved that it presents two contributions:

$$S_i^{Total}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l) = S_i^{neq}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l) + S_i(T_{EM},l)$$

with:

$$S_i^{neq}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l) = \frac{1}{4\pi i}\int_{-\infty+ia}^{+\infty+ia}\frac{d\Omega}{2\pi}\int_{-\infty+ia}^{+\infty+ia}\frac{d\Omega'}{2\pi}\frac{e^{-i(\Omega-\Omega')t}}{\Omega'}\epsilon_{ijk}\,\epsilon_{klm}\left[\partial'_l\langle E_j(\Omega,\mathrm{x})E_m(-\Omega',\mathrm{x}')\rangle\right]\Big|_{\mathrm{x}'=\mathrm{x}}$$
$$S_i(T_{EM},l) = -\frac{1}{4\pi}\int_{-\infty+ia}^{+\infty+ia}\frac{d\Omega}{2\pi}\,e^{-i\Omega t}\,\epsilon_{ijk}\langle E_j(\Omega,\mathrm{x})B_k(t_0,\mathrm{x})\rangle$$

The same analysis on the full non-equilibrium aspects (***dynamical*** and ***thermodynamical***) can be done, proving clearly that:

$$S^{neq}_{s,s'^*}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l) = \sum_{m=1}^{2}\left(S^{(X)}_{s,s'^*}(T^{(m)}_X,l) + S^{(P)}_{s,s'^*}(T^{(m)}_P,l)\right) + S_{s,s'^*}(T_{EM},l)$$

and:

$$S^{neq}_{s,s'^*}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l) = S^\infty_s(T^{(1)}_X,T^{(2)}_X,l) + S^{Trans}_{s,s'^*}(T^{(1)}_{X,P},T^{(2)}_{X,P},T_{EM},l)$$

[1] M. Antezza, L. P. Pitaevskii, S. Stringari, V. B. Svetovoy, Phys. Rev. A **77**, 022901 (2008).
[2] R. O. Behunin, B. L. Hu, Phys. Rev. A **84**, 012902 (2011).
[3] E. Calzetta, A. Roura, E. Verdaguer, Physica A **319**, 188 – 212 (2003).