

# **Thermal Casimir force, including applications to graphene**

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# CONTENT

- 1. Introduction**
- 2. Experiments with metals**
- 3. Experiments with dielectrics and semiconductors**
- 4. Casimir-Polder interaction of atoms with graphene**
- 5. Casimir interaction of graphene with material plate**
- 6. Conclusions and discussion**

# 1. INTRODUCTION

**Problems in the Lifshitz theory:**

**a) Violation of the Nernst heat theorem**

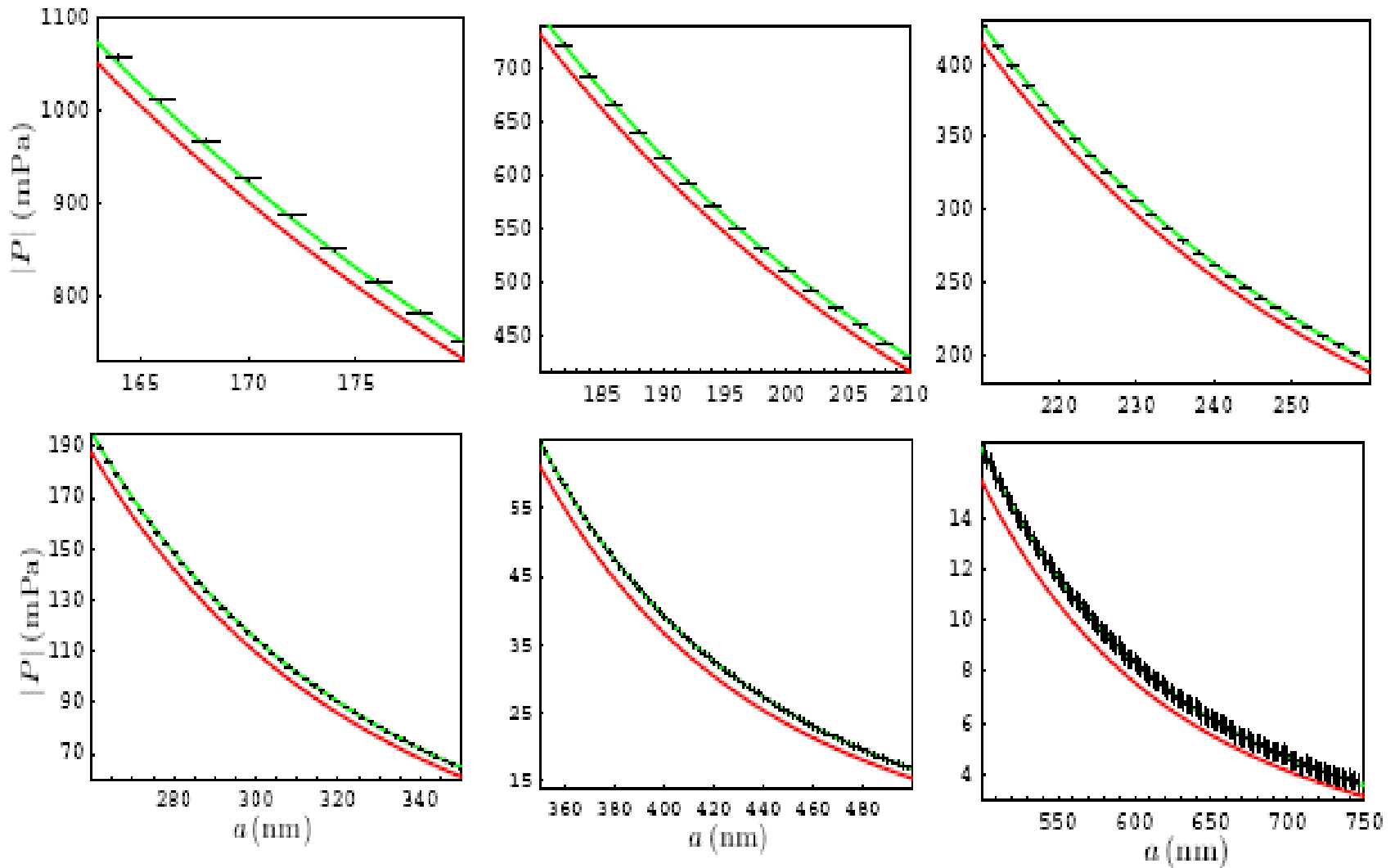
**b) Contradiction with the experimental data**

## 2. EXPERIMENTS WITH METALS

2a) Experiments with a micromachined oscillator:  
Au-coated sphere above Au-coated plate

$$|P(a, T)| = \frac{1}{2\pi R} \frac{\partial F(a, T)}{\partial a}$$

Decca, Lopez, Fischbach, Klimchitskaya, Krause, Mostepanenko,  
Ann. Phys. 2005; Phys. Rev. D 2007; Eur. Phys. J. C 2007



**Experimental crosses are shown at a 95% confidence level.**

**Red lines show the Drude model approach.**

**Green lines show the plasma model approach.**

# Claims against the comparison between experiment and theory:

- influence of surface roughness;
- error in absolute separations;
- errors in the optical data of Au;
- errors in the PFA;
- influence of patch potentials.

# Models of patches:

- Speake and Trenkel, Phys. Rev. Lett. 2003
- Behunin, Intravaia, Dalvit, Maia Neto, Reynaud, Phys. Rev. A 2012

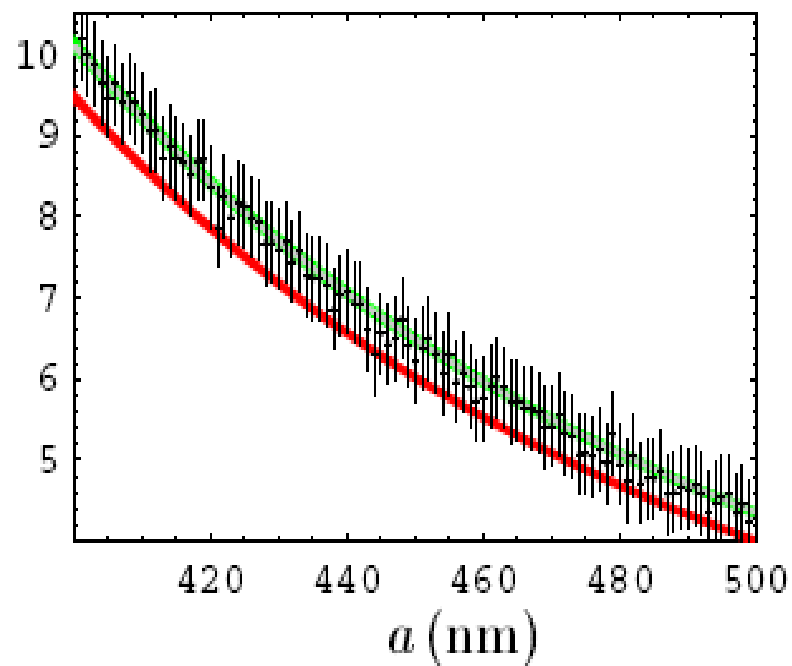
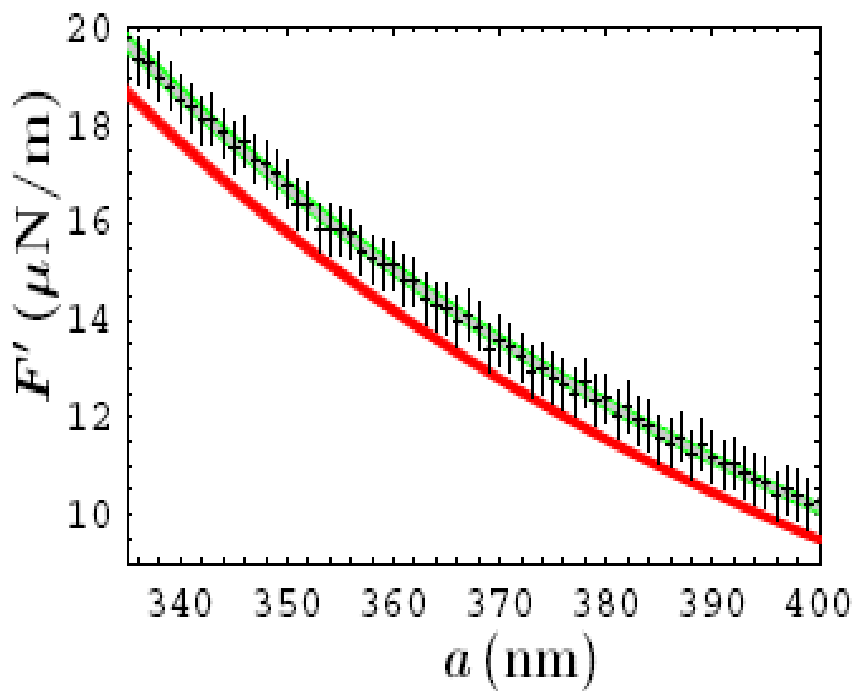
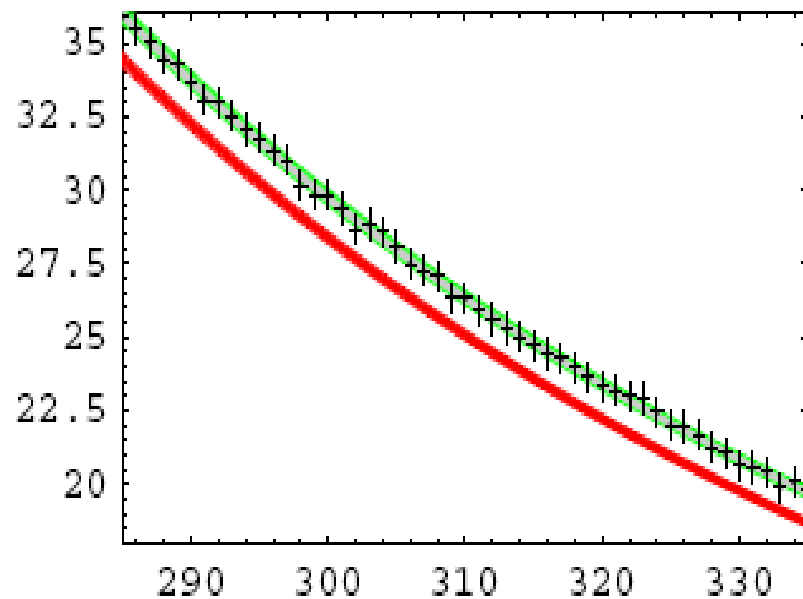
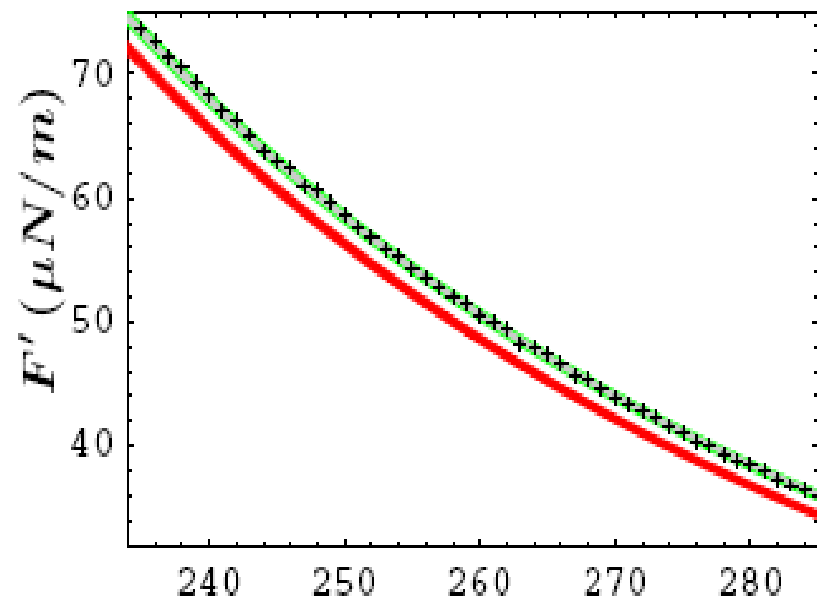
**Could the patch effect bring the data in agreement with the Drude model approach?**

**2b) Experiments with a dynamic AFM:  
Au-coated sphere above Au-coated plate**

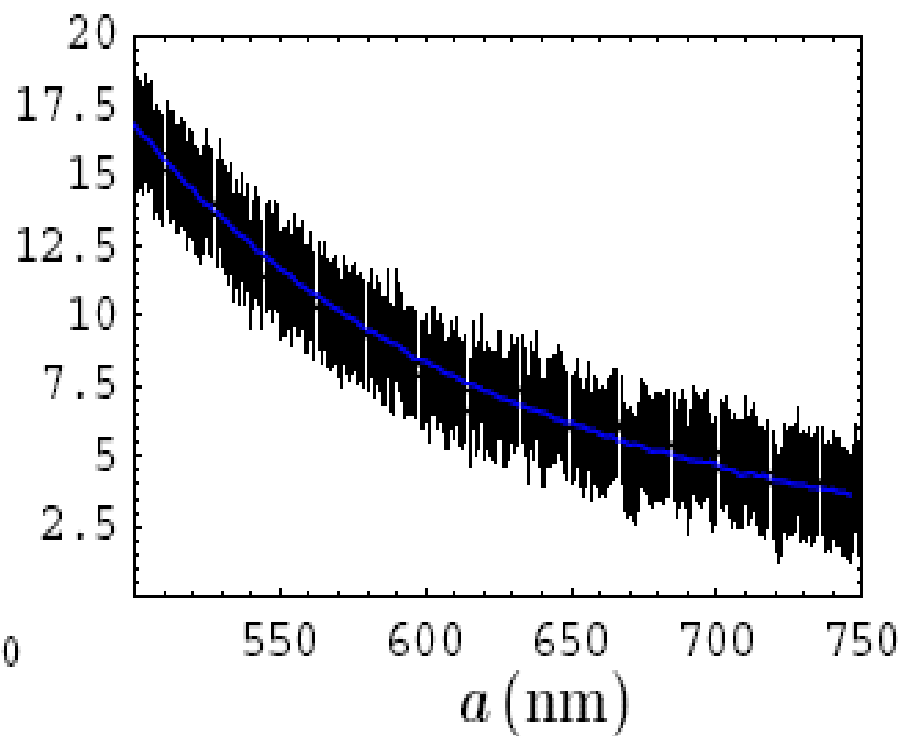
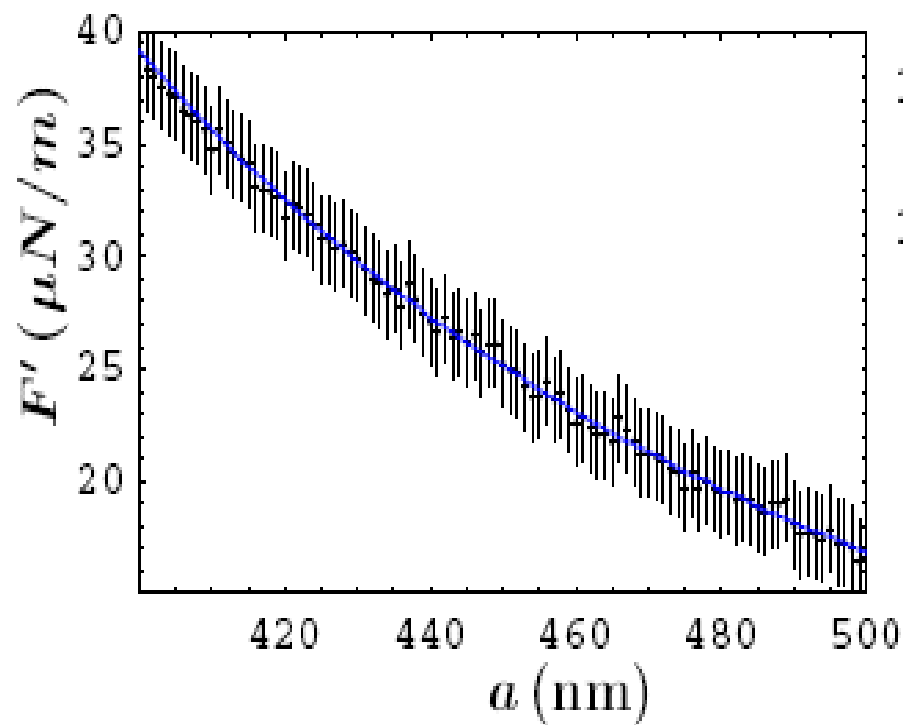
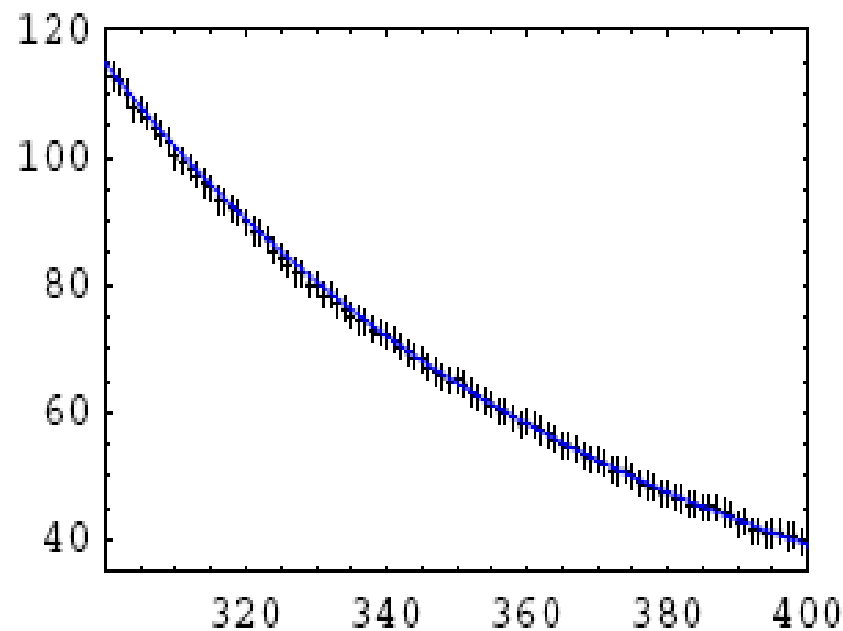
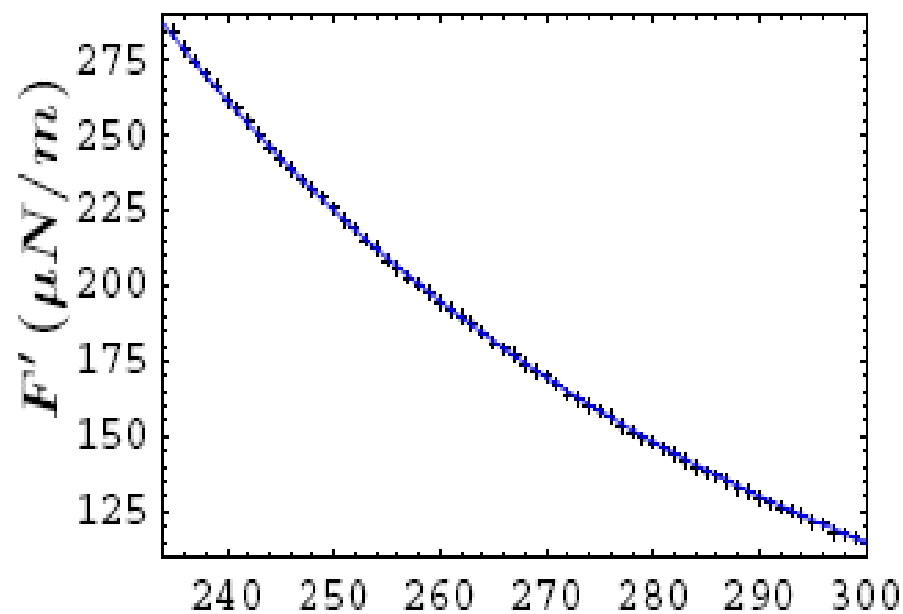
$$F' \equiv \frac{\partial F(a, T)}{\partial a} = -X'(a, R)(V_i - V_0)^2 - \frac{2k}{\omega_0} \Delta\omega$$

**Chang, Banishev, Castillo-Garza, Klimchitskaya, Mostepanenko,  
Mohideen, Phys. Rev. B 2012; Int. J. Mod. Phys.: Conf. Ser. 2012**





**Experimental crosses are shown at a 67% confidence level.**

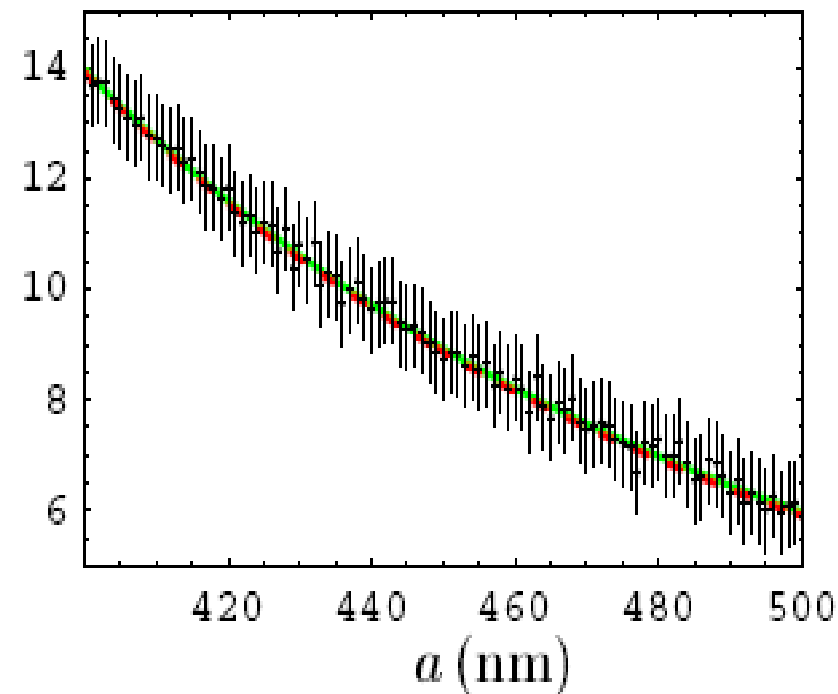
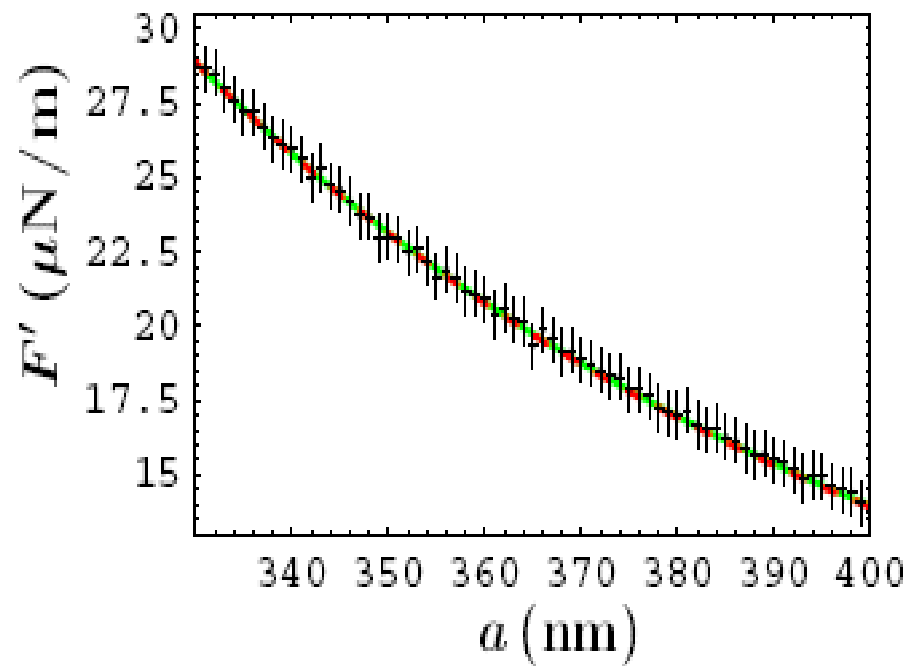
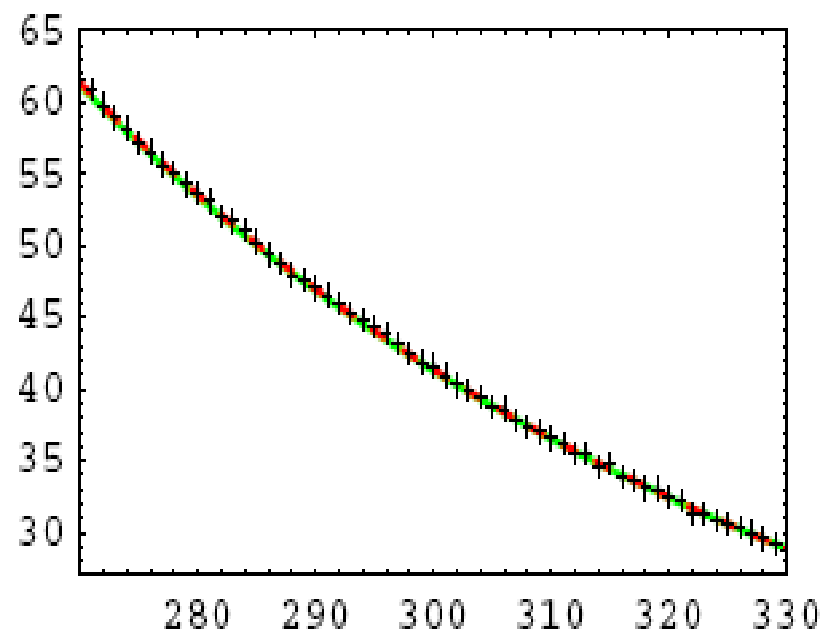
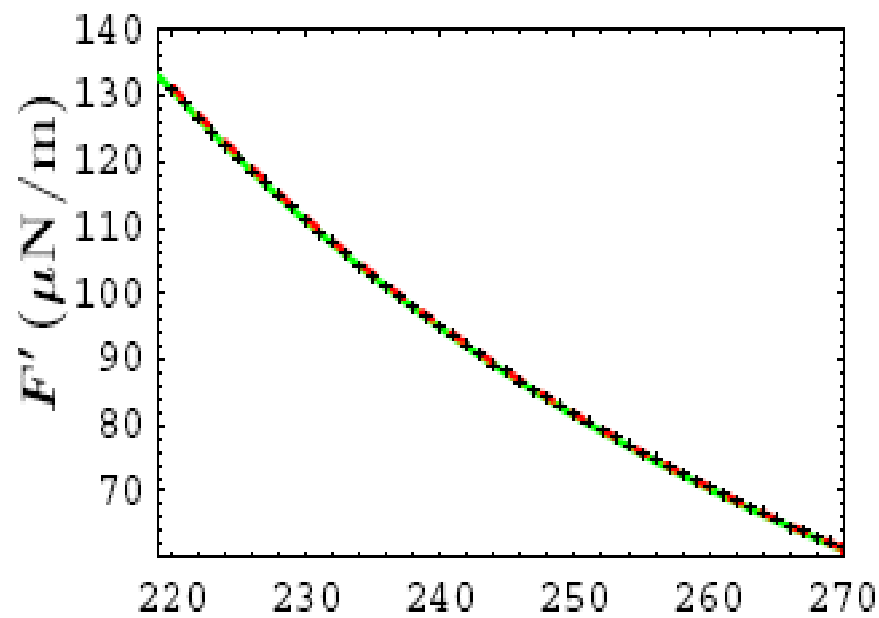


## 2c) Experiments with a dynamic AFM: Au-coated sphere above Ni-coated plate

Magnetic properties influence the Casimir force only through the TE zero-frequency term of the Lifshitz formula.

$$r_{\text{TE},p}^{(\text{Ni})}(0, k_{\perp}) = \frac{\mu(0)ck_{\perp} - \sqrt{c^2k_{\perp}^2 + \mu(0)\omega_p^2}}{\mu(0)ck_{\perp} + \sqrt{c^2k_{\perp}^2 + \mu(0)\omega_p^2}}$$

**Geyer, Klimchitskaya, Mostepanenko, Phys. Rev. B 2010;**  
**Banishev, Chang, Klimchitskaya, Mostepanenko, Mohideen,**  
**Phys. Rev. B 2012**

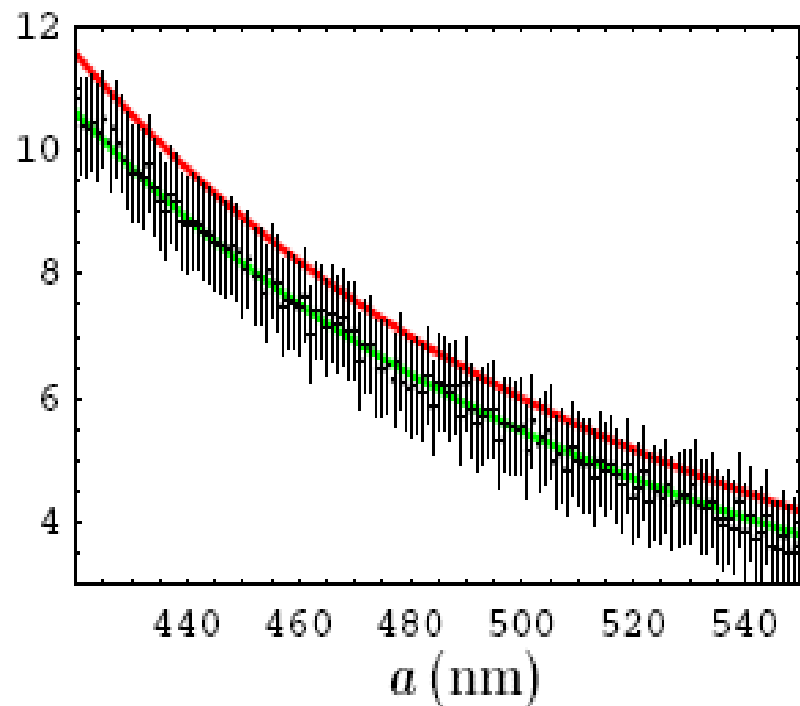
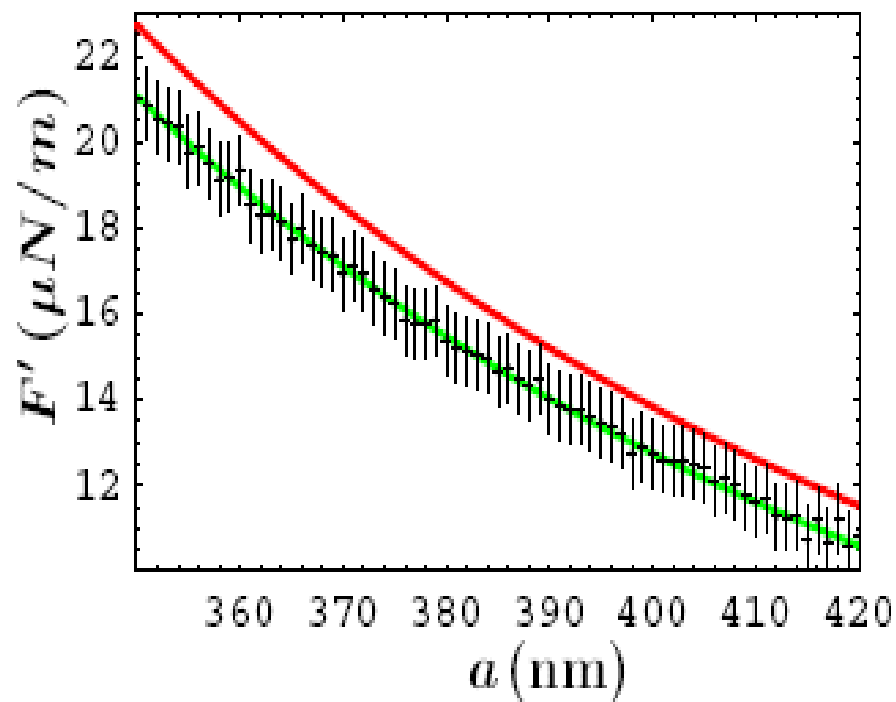
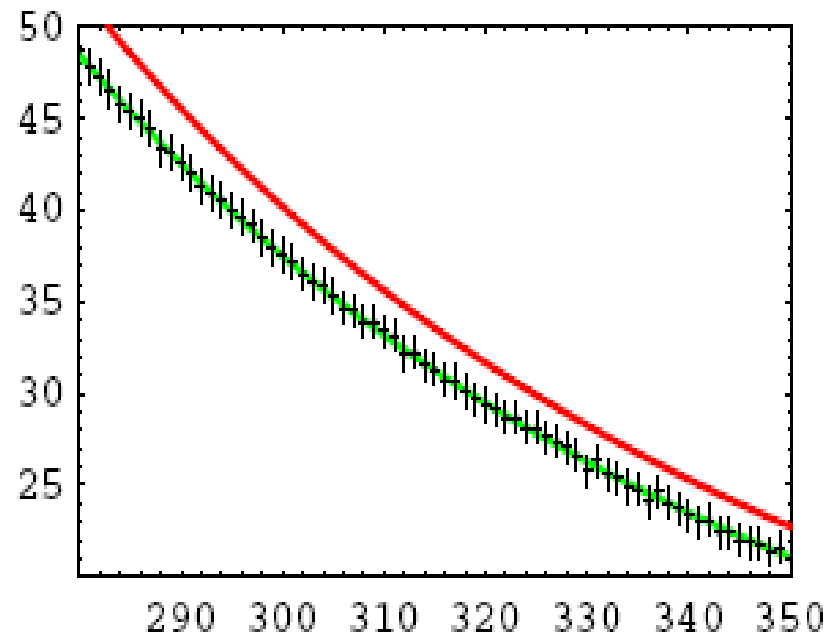
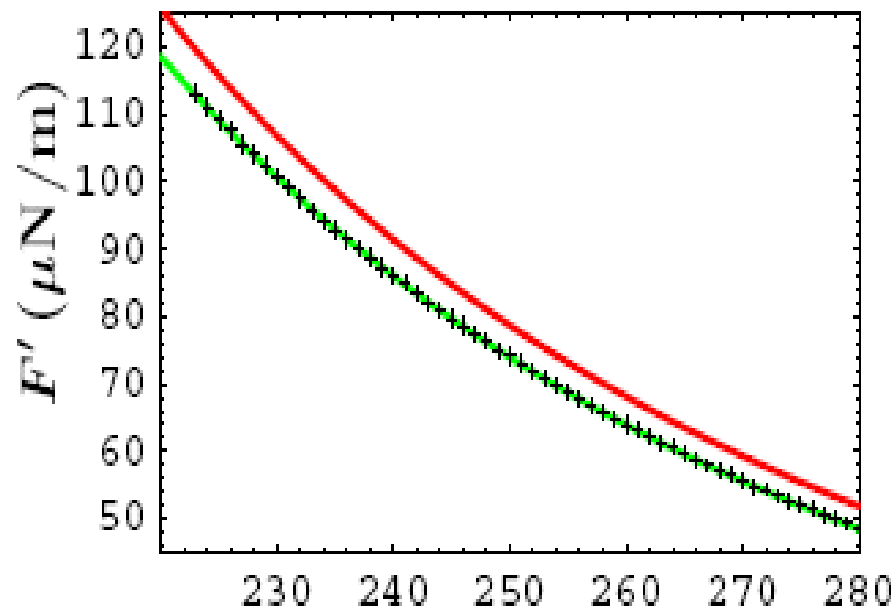


## 2d) Experiments with a dynamic AFM: Ni-coated sphere above Ni-coated plate

Magnetic properties influence the Casimir force when using both the plasma model and the Drude model approaches

$$r_{\text{TE,D}}^{(\text{Ni})}(0, k_{\perp}) = \frac{\mu(0) - 1}{\mu(0) + 1}$$

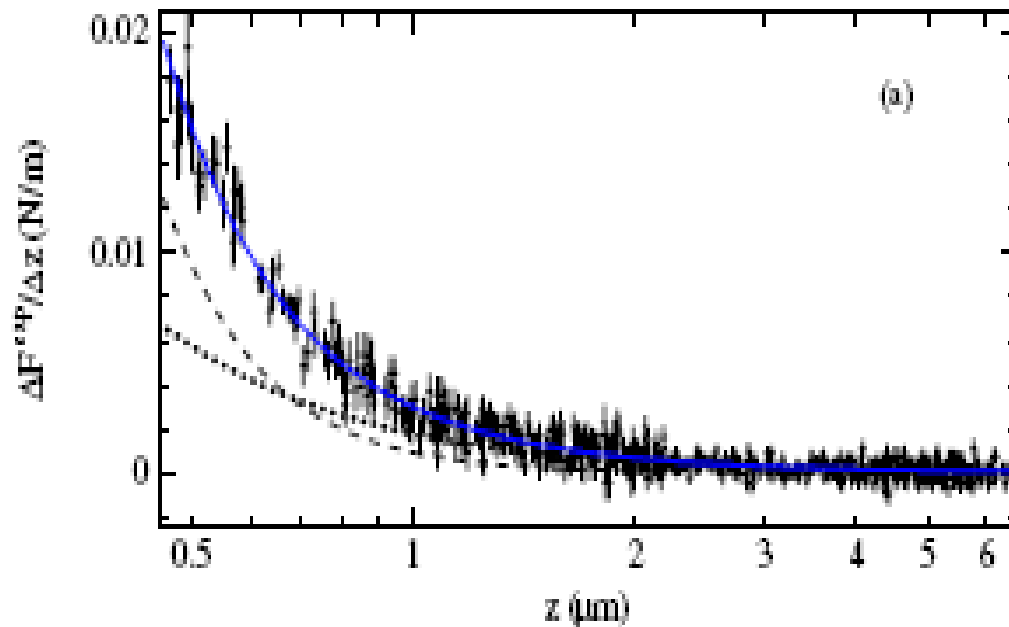
**Banishev, Klimchitskaya, Mostepanenko, Mohideen, in preparation**



## 2e) Torsion balance experiments

### Confirmation of the thermal correction predicted for ideal metals

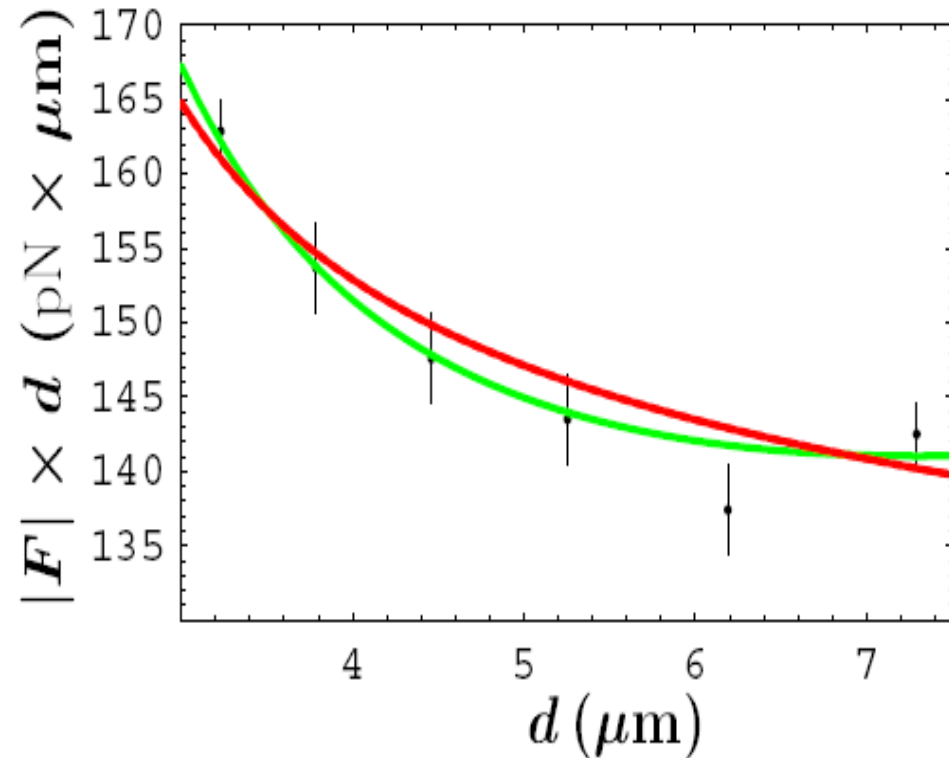
$$R = 20.7 \text{ cm}, \quad 0.48 \mu\text{m} \leq d \leq 6.5 \mu\text{m}$$



Masuda, Sasaki,  
Phys. Rev. Lett. 2009

$$\chi_{\min}^2 = 513, \quad f = 558, \quad P(\chi^2 > \chi_{\min}^2) = 0.91$$

# Sushkov, Kim, Dalvit, Lamoreaux, Nature Phys. 2011



Klimchitskaya, Bordag, Fischbach,  
Krause, Mostepanenko,  
Int. J. Mod. Phys. A 2011;  
Klimchitskaya, Bordag,  
Mostepanenko, Int. J. Mod. Phys. A  
2012

**f=4**

**Fit using the Drude model:**

$$\chi_{\min}^2 = 19.76, \quad P(\chi^2 > \chi_{\min}^2) = 0.16$$

$$V_{\text{rms}} = 5.4 \text{ mV}, \quad a = -3 \text{ pN}$$

**Fit using the plasma model:**

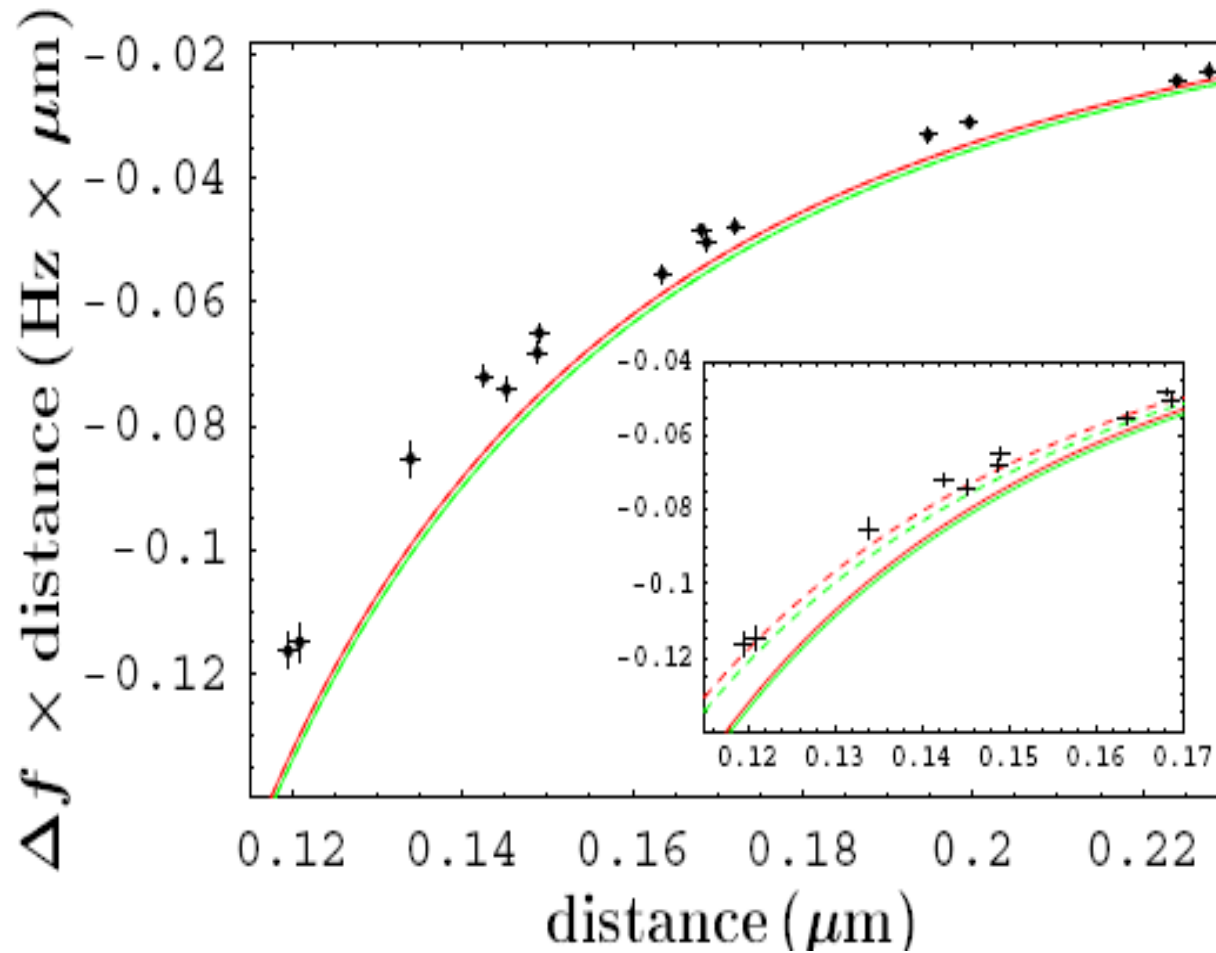
$$\chi_{\min}^2 = 2.76, \quad P(\chi^2 > \chi_{\min}^2) = 0.61$$

$$V_{\text{rms}} = 4.5 \text{ mV}, \quad a = 3.6 \text{ pN}$$



## 2f) Experiment using a microresonator

Garcia-Sanches, Fong, Bhaskaran, Lamoreaux, Tang, Phys. Rev. Lett. 2012



$$\chi_D^2 > 300$$

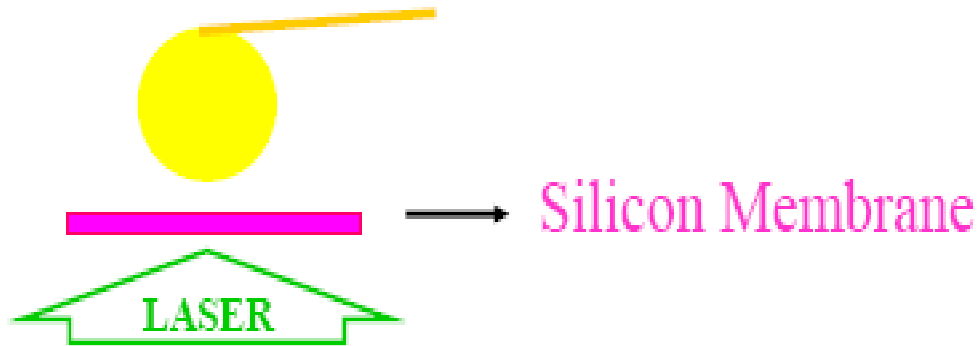
$$\chi_p^2 > 419$$

$$P < 10^{-8}$$

Bordag, Klimchitskaya, Mostepanenko, arXiv:1208.1757v1

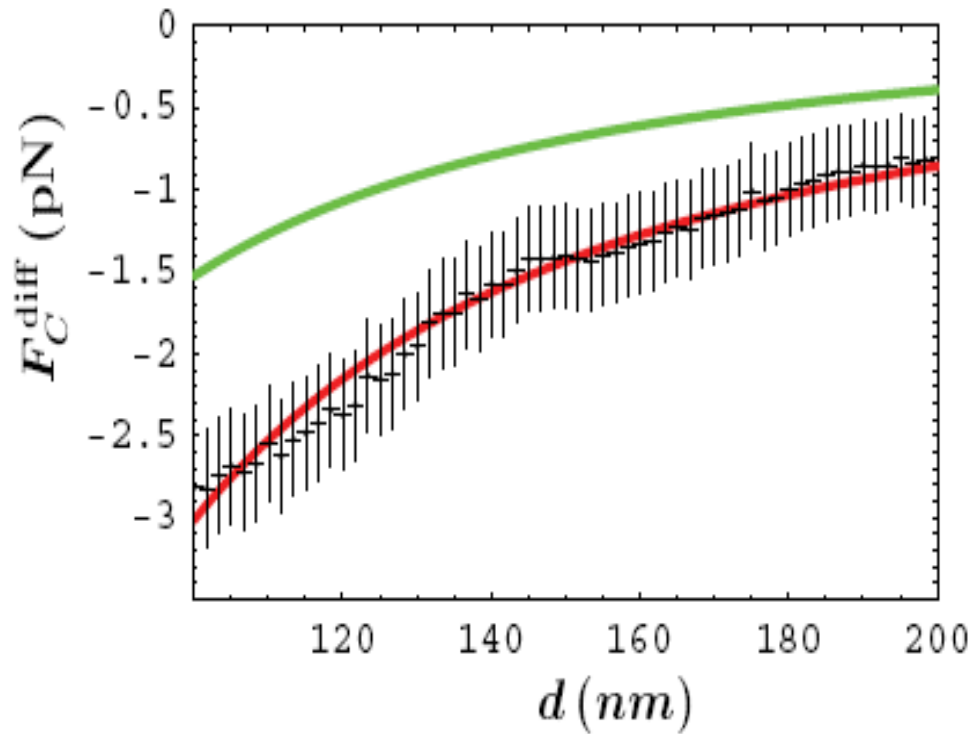
### 3. EXPERIMENTS WITH DIELECTRICS AND SEMICONDUCTORS

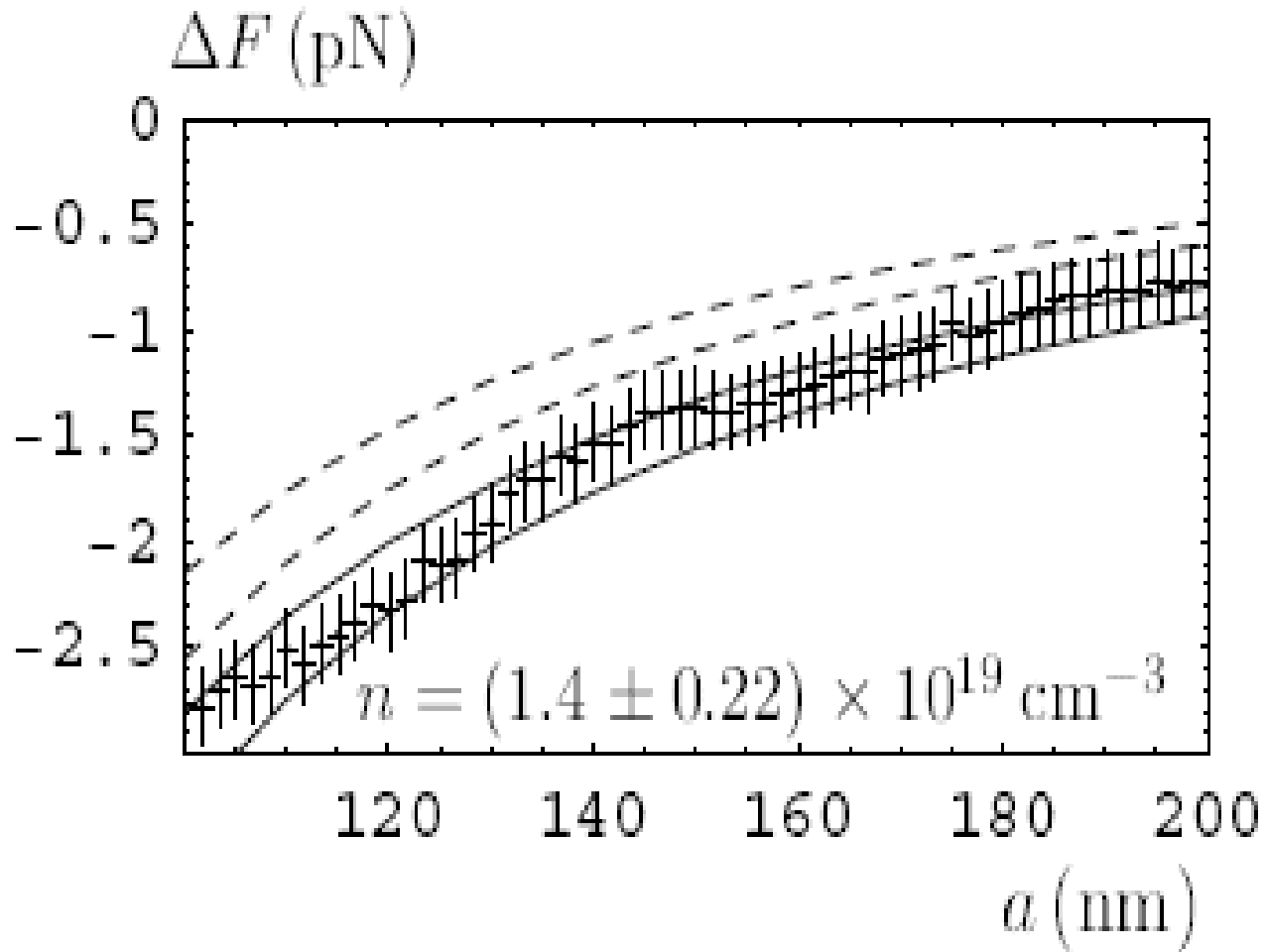
#### 3a) Optical modulation of the Casimir force: Au-coated sphere above Si plate



Chen, Klimchitskaya, Mostepanenko, Mohideen,  
Opt. Express 2005; Phys. Rev. B 2007;  
Klimchitskaya, Mostepanenko, Mohideen, J. Phys. A 2008

# Difference of the Casimir forces in the presence and absence of laser light

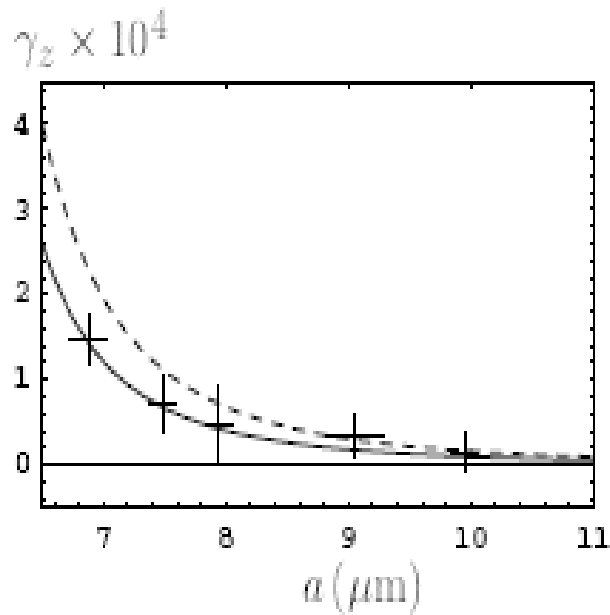




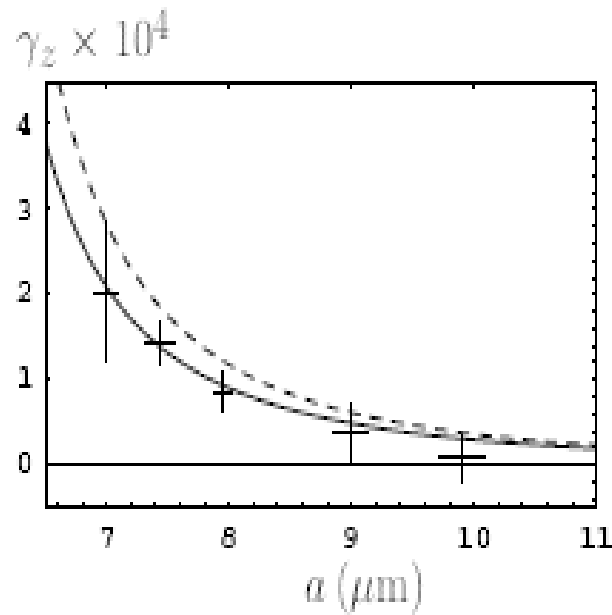
Pitaevskii, Phys. Rev. Lett. 2008

Geyer, Klimchitskaya, Mohideen, Mostepanenko, Phys. Rev. Lett. 2009

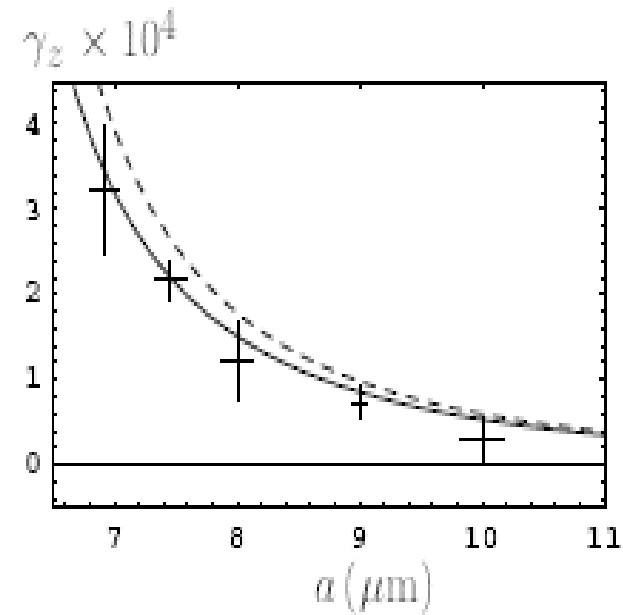
### 3b) Relative frequency shift of center-of-mass oscillations of Bose-Einstein condensate



$$T_p = T_e = 310 \text{ K}$$



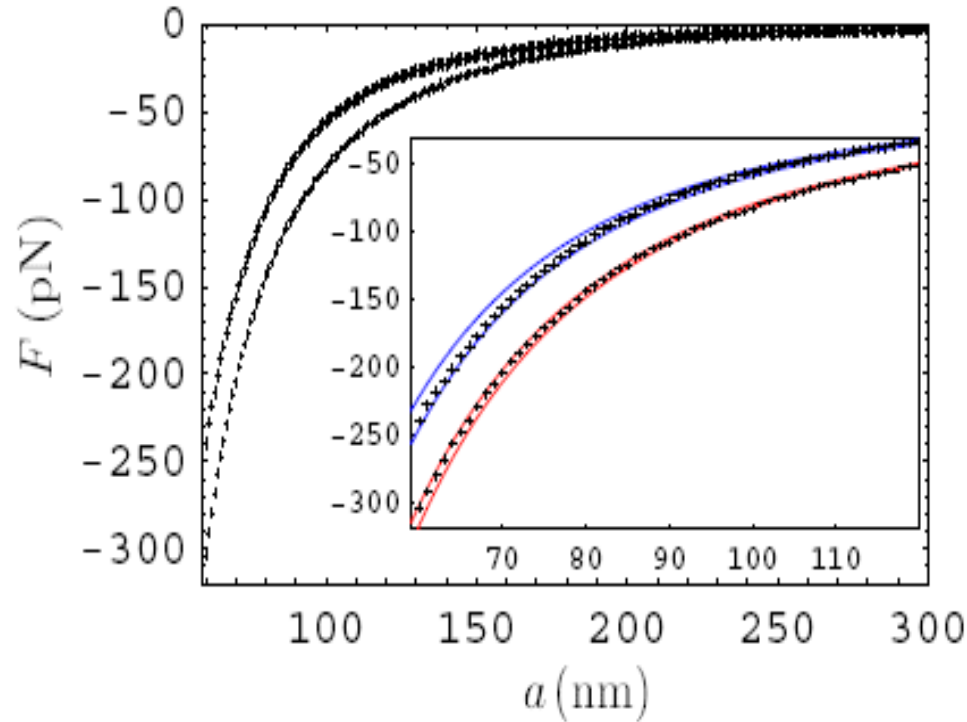
$$T_p = 479 \text{ K}$$



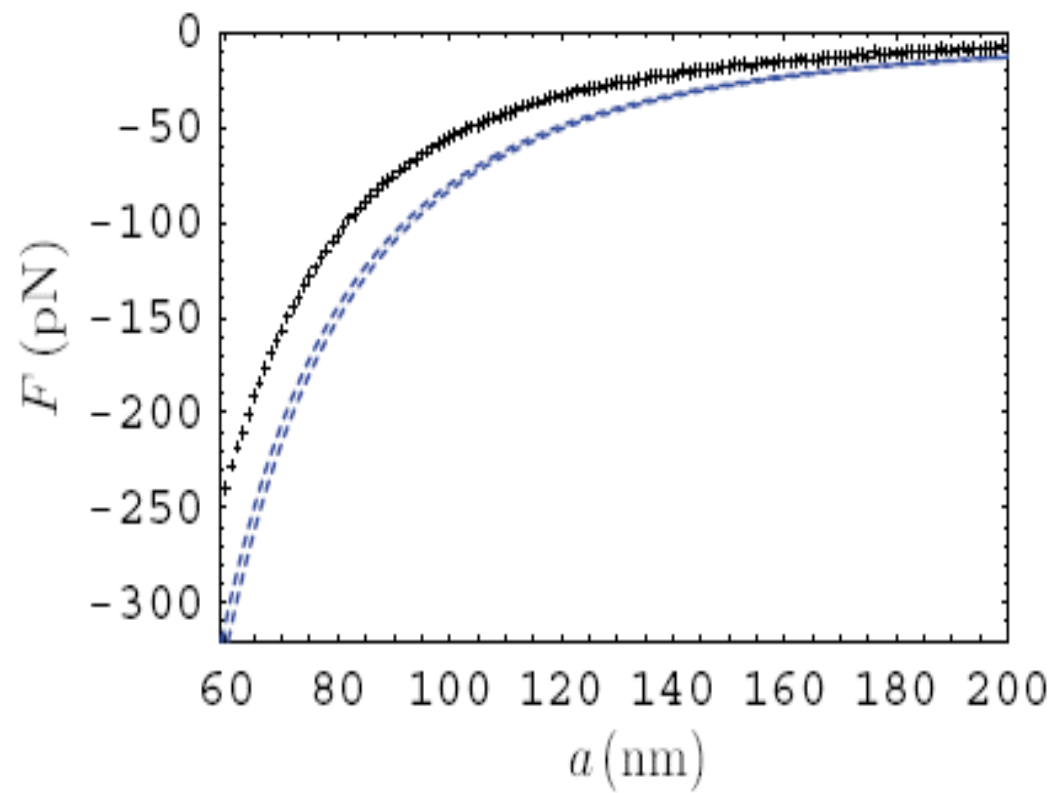
$$T_p = 605 \text{ K}$$

Obrecht, Wild, Antezza, Pitaevskii, Stringari, Cornell, PRL (2007);  
Klimchitskaya, Mostepanenko, JPA (2008).

### 3c) The Casimir force between Au sphere and ITO plate



Chang, Banishev, Klimchitskaya, Mostepanenko, Mohideen, Phys. Rev. Lett. 2011;  
Banishev, Chang, Castillo-Garza, Klimchitskaya, Mostepanenko, Mohideen,  
Phys. Rev. B 2012; Int. J. Mod. Phys. A 2012



## 4. CASIMIR-POLDER INTERACTION OF ATOM WITH GRAPHENE

Dirac model of graphene

$$r_{\text{TM}}^{(g)}(i\xi_l, k_{\perp}) = \frac{q_l \Pi_{00}}{q_l \Pi_{00} + 2\hbar k_{\perp}^2}$$

$$r_{\text{TE}}^{(g)}(i\xi_l, k_{\perp}) = -\frac{k_{\perp}^2 \Pi_{\text{tr}} - q_l^2 \Pi_{00}}{k_{\perp}^2 (\Pi_{\text{tr}} + 2\hbar q_l) - q_l^2 \Pi_{00}}$$

$$q_l^2 = k_{\perp}^2 + \frac{\xi_l^2}{c^2}$$

**Bordag, Fialkovsky, Gitman, Vassilevich, Phys. Rev. B 2009;**  
**Fialkovsky, Marachevsky, Vassilevich, Phys. Rev. B 2011**



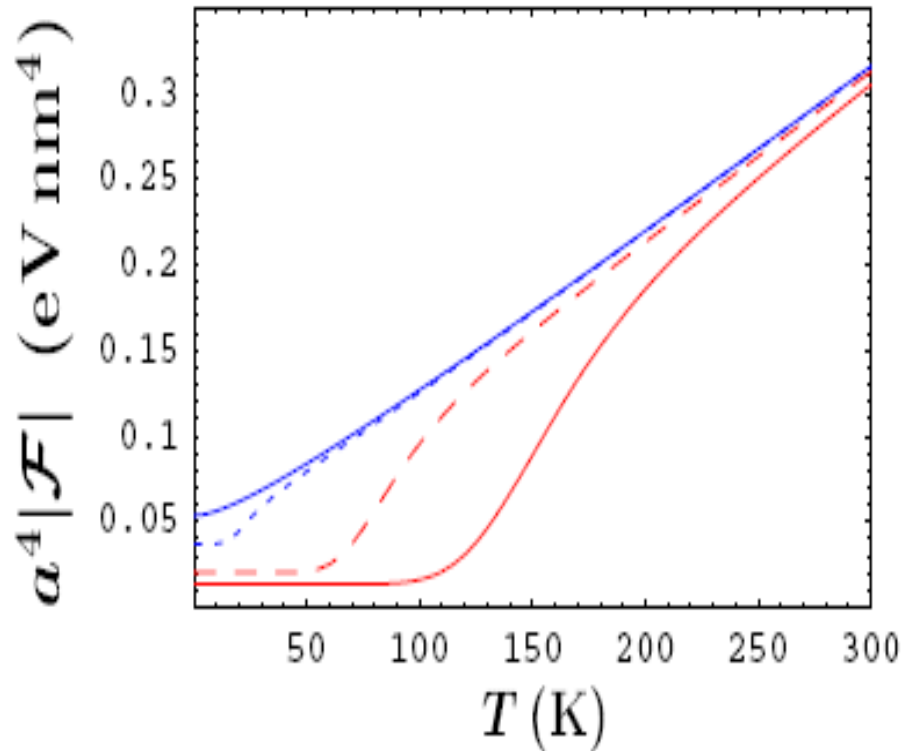
## Hydrodynamic model of graphene

$$r_{\text{TM}}^{(g)}(i\xi_l, k_{\perp}) = \frac{c^2 q_l K}{c^2 q_l K + \xi_l^2}$$

$$r_{\text{TE}}^{(g)}(i\xi_l, k_{\perp}) = -\frac{K}{K + q_l}$$

$$K = 2\pi \frac{ne^2}{mc^2} = 6.75 \times 10^5 \text{ m}^{-1}$$

## Metastable He interacting with graphene



Lines from the lowest to the highest are for

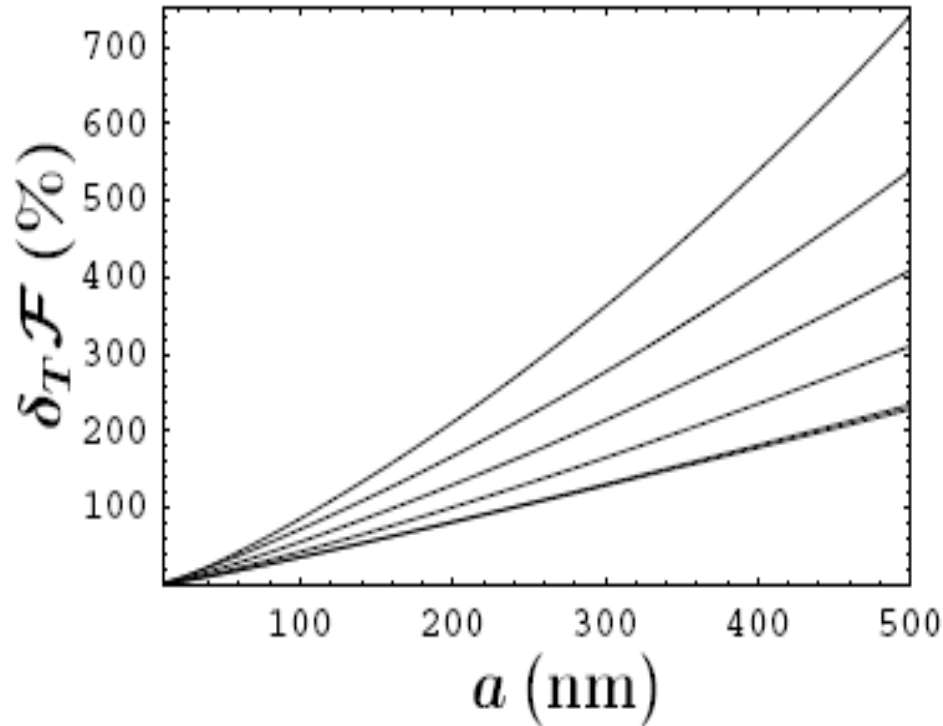
$$\Delta = 0.1 \text{ eV}$$

$$\Delta = 0.05 \text{ eV}$$

$$\Delta = 0.01 \text{ eV}$$

$$\Delta = 0 \text{ eV}$$

## Relative thermal correction



Lines from the highest to the lowest are for

$$\Delta = 0.1 \text{ eV}$$

$$\Delta = 0.05 \text{ eV}$$

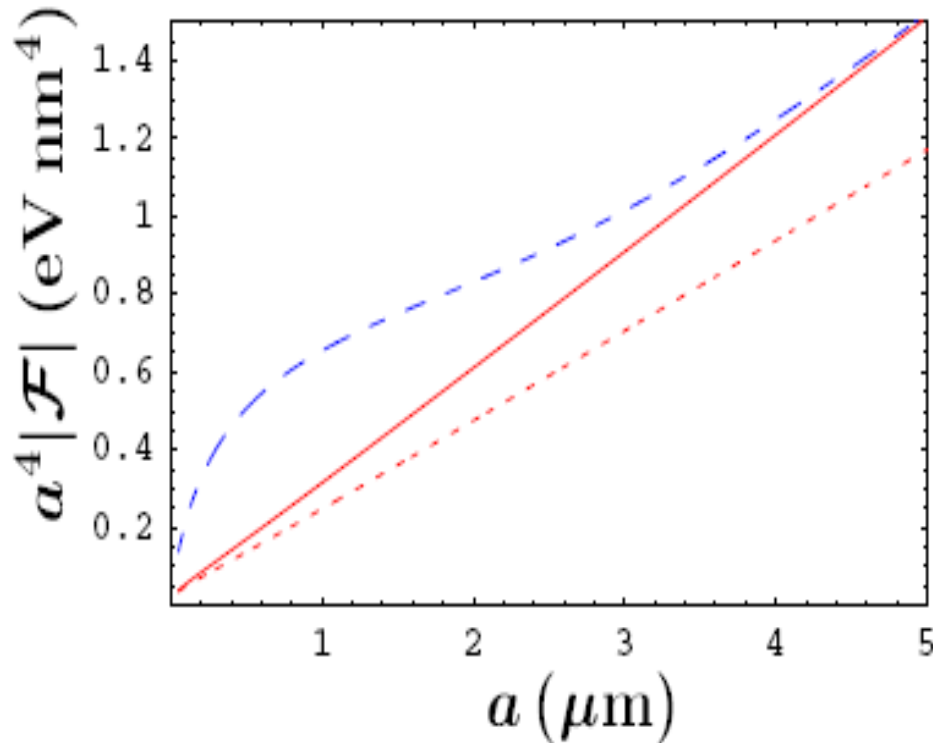
$$\Delta = 0.025 \text{ eV}$$

$$\Delta = 0.01 \text{ eV}$$

$$\Delta \leq 0.001 \text{ eV}$$

$$T = 300 \text{ K}$$

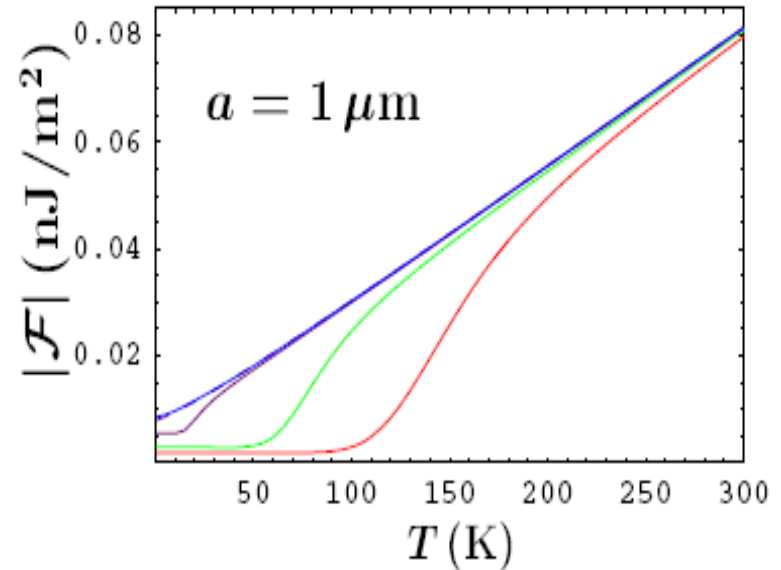
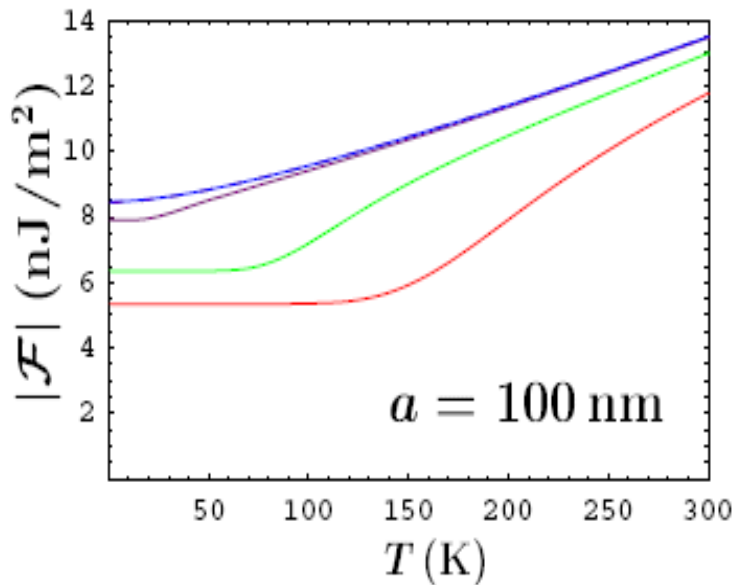
## Comparison between hydrodynamic (dashed line) and Dirac models



$T = 300 \text{ K}$

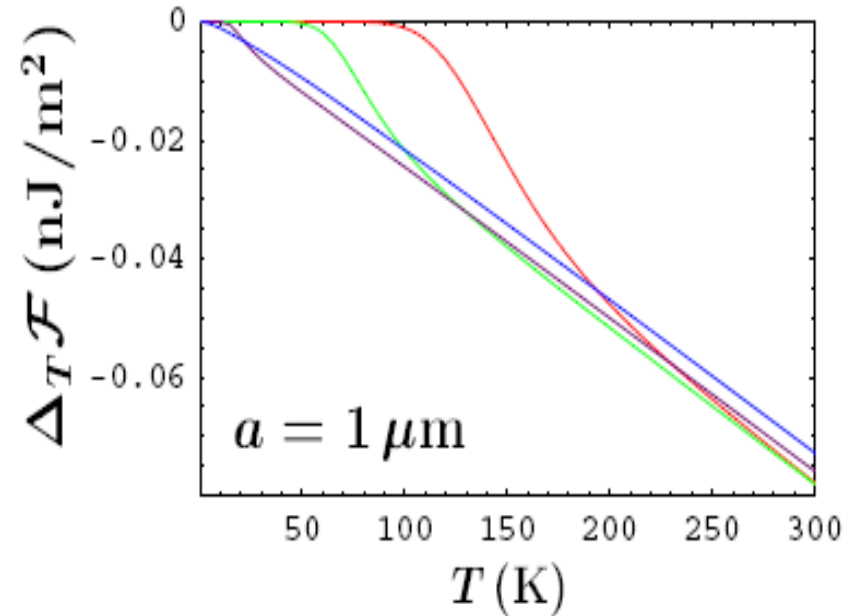
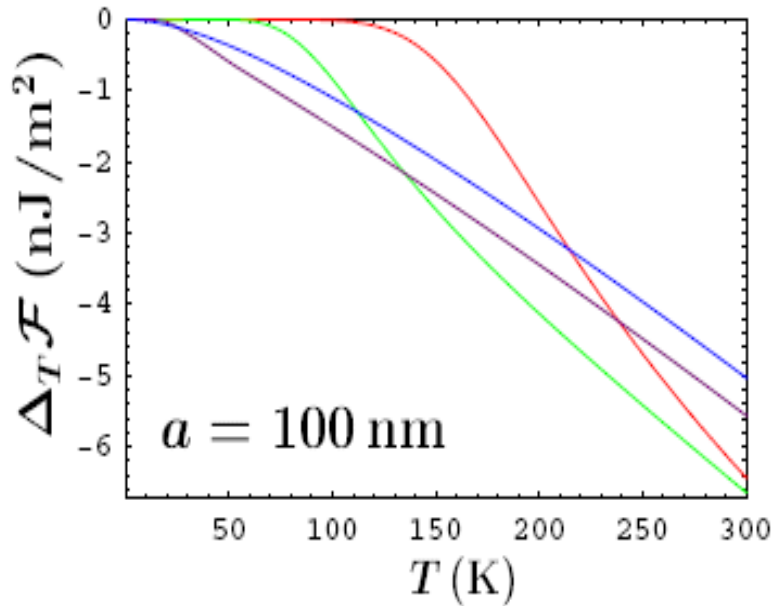
# 5. CASIMIR INTERACTION OF GRAPHENE WITH MATERIAL PLATE

Graphene interacting with Si plate



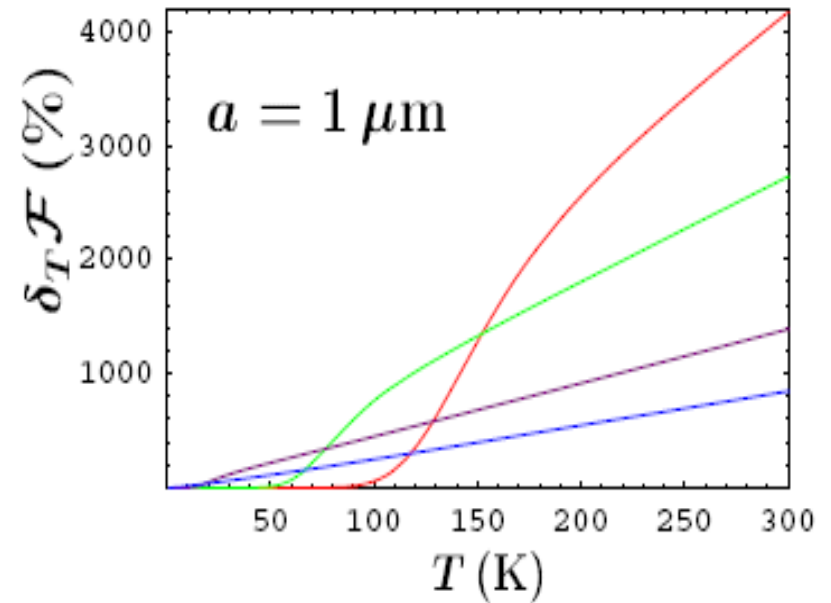
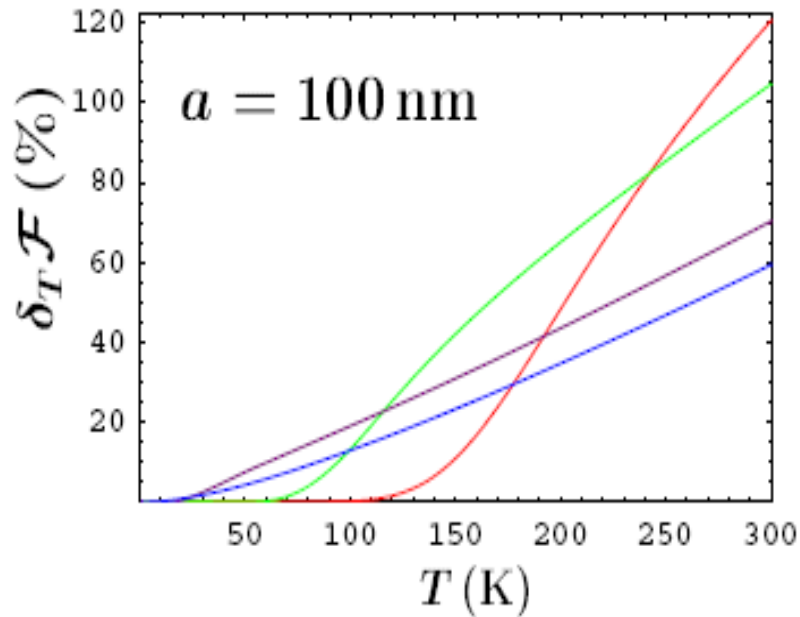
$\Delta = 0.1$  eV (red line)       $\Delta = 0.05$  eV (green line)  
 $\Delta = 0.01$  eV (pink line)     $\Delta \leq 0.001$  eV (blue line)

## Thermal correction to the Casimir energy



$\Delta = 0.1$  eV (red line)       $\Delta = 0.05$  eV (green line)  
 $\Delta = 0.01$  eV (pink line)     $\Delta \leq 0.001$  eV (blue line)

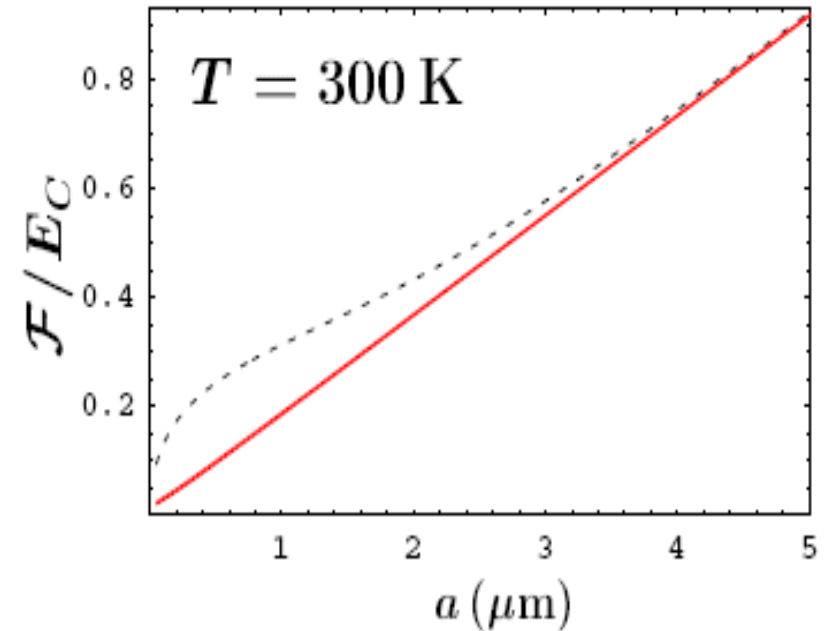
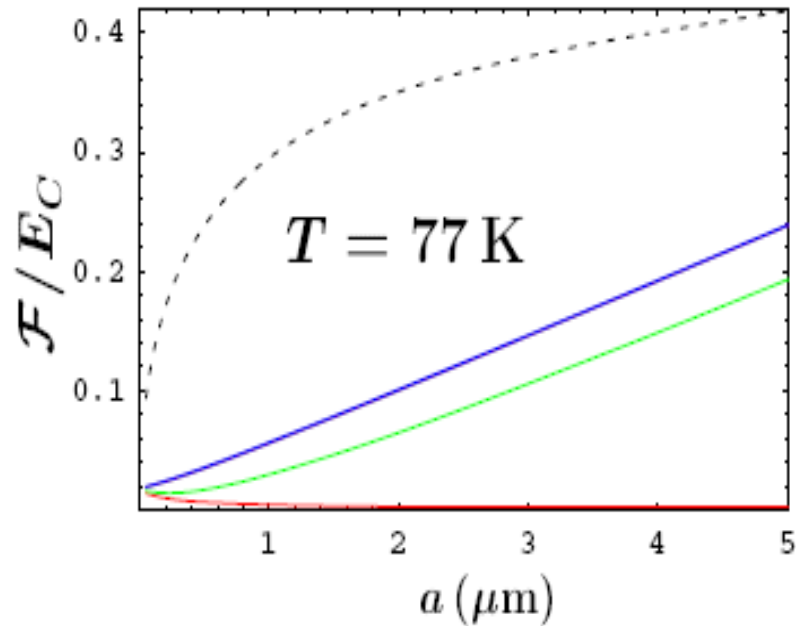
## Relative thermal correction



$\Delta = 0.1$  eV (red line)       $\Delta = 0.05$  eV (green line)

$\Delta = 0.01$  eV (pink line)       $\Delta \leq 0.001$  eV (blue line)

## Comparison between hydrodynamic (dashed line) and Dirac models





## 6. CONCLUSIONS AND DISCUSSION

- a) Many experiments performed by different experimental groups with metallic, dielectric and semiconductor test bodies demonstrate that the Lifshitz theory using the most realistic models of dielectric permittivity is in deep contradiction with the measurement data for the thermal Casimir force.**

**b) The same measurement data are consistent with the Lifshitz theory when the relaxation properties of conduction electrons are omitted for metals and the free charge carriers are disregarded for dielectrics.**

**c) All kinds of possible weaknesses in these experiments and in their comparison with theory discussed in the literature were carefully analyzed by different authors and shown to have no effect on the conclusions obtained.**

**d) The two experiments claiming agreement with the Drude model approach for metals are not independent measurements. They are based on the fitting procedures between the data and some model expressions for hypothetical contributions to the force. There are serious specific objections against comparison of the measurement data with theory in these experiments.**

**e) Measurement of the Casimir-Polder and Casimir interactions of atoms and material bodies with graphene opens new prospective opportunities for investigation of large thermal effects at short separations. There is no direct analogy, however, between theoretical predictions obtained using the Drude model approach for metals and the Dirac model for graphene.**