The influence of a surface on the interaction between two atoms

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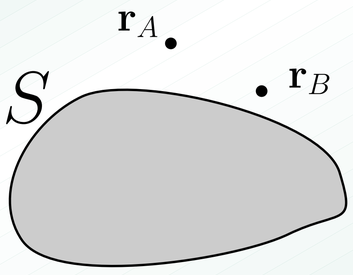
1 Introduction

One of the most striking features of the van der Waals interaction is its non-additivity[1]. It states that the force suffered by a body, A, due to two other bodies, B and C, is not the superposition of the forces A would felt in the presence of B and C alone. Although this is well-know for many years, quantitative predictions of the non-additivity effects have been performed only in few cases. Recently, C. Eberlein and R. Zietal proposed a method capable of evaluate the dispersive force of an atom close to a perfectly conducting surface of an arbitrary shape[2]-[4]. In this work we will generalize their method to two atoms and obtain a analytical expression allowing us to analyse quantitatively the effect an arbitrary perfectly conducting surface has on the interaction between two atoms.



2 The Eberlein-Zietal method

Consider a pollarizable atom without permanent dipole moment in the presence of a perfectly conducting grounded surface S. The vacuum zero-point fluctuations will induce a oscillating dipole in the atom that will interact with the surface. The exact calculation of such a force belongs to the quantum electrodynamics framework. However, if the atom is close enough to the surface (compared to its relevant wavelength transitions) we don't need to quantize the electromagnetic field, once it is, within an excellent approximation, instantaneous. The interaction between the atom and the surface is, in this regime, mediated by a classical field. Eberlein and Zietal has shown [2] that this force may be evaluated from the Hamiltonian of interaction

$$H_{int} = \frac{1}{2\varepsilon_0}(\mathbf{d}\cdot\nabla')(\mathbf{d}\cdot\nabla)G_H(\mathbf{r},\mathbf{r}')\Big|_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_0}, \quad (1)$$

where $\mathbf{d}$ is the atomic dipole operator and G_H is a function that satisfies

$$\nabla^2 G_H(\mathbf{r},\mathbf{r}') = 0$$

$$\left[\frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|} + G_H(\mathbf{r},\mathbf{r}')\right]_{\mathbf{r}\in S} = 0\,. \quad (2)$$

This is analogous to the equations satisfied by the potential generated by the images in the electrostatic problem of a charge in the presence of the surface S. This renders a simple and powerful way to solve our problem whenever the image method is applicable. An infinite plane, a sphere and a disc are some nice examples[5]-[6].

3 Generalization for two atoms

In this situation, the interaction Hamiltonian (1) becomes

$$H_{int} = H_A + H_B + H_{london} + H_{ABS}\,, \quad (3)$$

where

$$H_i = \frac{1}{2\varepsilon_0}(\mathbf{d}_i\cdot\nabla')(\mathbf{d}_i\cdot\nabla)G_H(\mathbf{r},\mathbf{r}')\Big|_{\mathbf{r}=\mathbf{r}'=\mathbf{r}_i}$$

$$H_{london} = \frac{1}{4\pi\varepsilon_0}(\mathbf{d}_B\cdot\nabla')(\mathbf{d}_A\cdot\nabla)\frac{1}{|\mathbf{r}-\mathbf{r}'|}\Big|_{\mathbf{r}=\mathbf{r}_A,\mathbf{r}'=\mathbf{r}_B}$$

$$H_{ABS} = \frac{1}{\varepsilon_0}(\mathbf{d}_B\cdot\nabla')(\mathbf{d}_A\cdot\nabla)G_H(\mathbf{r},\mathbf{r}')\Big|_{\mathbf{r}=\mathbf{r}_A,\mathbf{r}'=\mathbf{r}_B}. \quad (4)$$

In the above expression, H_i is the only term that would survive if we have only the atom i interacting with the surface S. H_{london} is the interaction between the atoms in the absence of any surface. In the next section we will show that it corresponds to the London energy of interaction, as expected in the non-retarded regime. Finally, H_{ABS} is the only term involving the three terms and accounts for the non-additivity in their interaction. This is the central term to our analysis and will be left to section 5. The great advantage of our procedure is now evident - it separates the non-additivity term from the others.

4 London's interaction energy

The term H_{london} is the only that survives in the absence of surface ($G_H = 0$). From now on we will, for the sake of simplicity, assume the atoms to have spherical symmetry. Employing the perturbation theory in second order, we obtain

$$E_{london} = -{\sum_{r,s}}' \frac{\langle 0\,,0|H_{london}|r,s\rangle\langle r,s|H_{london}|0\,,0\rangle}{E^A_{0r}+E^B_{0s}}$$

$$= -\frac{1}{24\pi^2\varepsilon_0^2R^6}{\sum_{r,s}}' \frac{|\mathbf{d}^{or}_A|^2|\mathbf{d}^{0s}_B|^2}{(E^A_{0r}+E^B_{0s})}. \quad (5)$$

This way we re-obtain the well-known result for the interaction between two atoms in the vacuum in the non-retarded regime.

5 Non-additivity

We will, now, focus on the term H_{ABS} in equation (4). As we have discussed, this terms contemplates quantitatively the non-additivity effects of our system. Working this term through perturbation theory we obtain a vanishing first order contribution. In second order, the non-vanishing terms involving H_{ABS} are

$$-2{\sum_{r,s}}' \frac{\langle 0\,,0|H_{london}|r,s\rangle\langle r,s|H_{ABS}|0\,,0\rangle}{E^A_{0r}+E^B_{0s}} \quad (6)$$

and

$$-{\sum_{r,s}}' \frac{\langle 0\,,0|H_{ABS}|r,s\rangle\langle r,s|H_{ABS}|0\,,0\rangle}{E^A_{0r}+E^B_{0s}}. \quad (7)$$

After some calculation, we obtain

$$\frac{E_{NA}}{E_{london}} = \frac{3\pi R^3}{4}\left\{\partial_i\partial'_i G_H - 3\hat{R}_i\hat{R}_j\partial_i\partial'_j G_H\right\}_{\mathbf{r}=\mathbf{r}_A,\mathbf{r}'=\mathbf{r}_B}$$

$$+\frac{8\pi^2R^6}{3}\left\{\partial_i\partial'_j G_H\right\}^2_{\mathbf{r}=\mathbf{r}_A,\mathbf{r}'=\mathbf{r}_B}. \quad (8)$$

This way we have a quantitative measure of the effect presented by the surface S on the interaction between the two atoms. It is remarkable that this ratio is independent on the internal structure of the atoms (remember that we are supposing them spherically symmetrical). With this expression we may analyse the non-additivity effects for any grounded surface. We could also analyse the influence of one atom on the interaction between the surface with the other atom. However, since the atom-surface interaction has, in general, a first order contribution, the ratio between the non-additivity term and the free atom-surface interaction will be much smaller than (8).

6 Conclusion

In this poster we have shown how to evaluate analytically the non-additivity effects for a system composed by two atoms and a perfectly conducting grounded surface. Non-additivity effects to this type of system have already been treated, [7]-[8], however, as long as the authors know, only for some particular geometry. Our treatment has the advantage of be applicable to an arbitrary surface.

As a perspective, we shall now deal with a particular example of great interest, namely, two atoms in the presence of a sphere.

Acknowledgments

This work has been partially supported by CNPq. The authors thank Paulo A. Maia Neto for valuable discussions.

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