Consistent Perturbative Electromagnetic Roughness Corrections to Casimir Energies

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Grant no. PHY-09-02054

We develop a systematic field theoretic description for perturbative roughness corrections to the Casimir energy of parallel plates. Roughness is modeled by prescribing a generating functional for correlation functions of the height profile. The model is derived from the coupling of electrodynamic quantum fields in four-dimensional Euclidean space-time to the height profile on the surface of the plate. In the perturbative regime our theory shows a decreasing magnitude of the Casimir force as roughness increases. This is consistent with expectations from multiple scattering theory. Contrary to previous perturbative estimates, the roughness corrections to the Casimir force in our theory are finite for any physically realizable roughness correlation function and not restricted to Gaussian correlations. Perturbative roughness corrections are found to be bounded above by the PFA and also below. Compared to previous approaches, perturbative roughness corrections are relatively and depend only weakly on the correlation length of the profile.

The UV problem of Roughness Corrections

➢Perturbative corrections to the electromagnetic Casimir energy of parallel plates due to their roughness are of the general form:

$$\Delta\mathcal{E}_{Rough} = \int \frac{d\mathbf{k}}{(2\pi)^2} G(\mathbf{k};a,\varepsilon) D(\mathbf{k})$$

where $G(\mathbf{k};a,\varepsilon)$ is a loop-function that depends on the separation a of the plates and their dielectric permittivity $\varepsilon(\omega)$ and

$$D(\mathbf{k}) = \int e^{i\mathbf{k}\mathbf{x}} \langle h(\mathbf{0}) h(\mathbf{x})\rangle d\mathbf{x} \quad \Rightarrow \quad - - - - -$$

is the correlation function of the roughness profile $h(\mathbf{x})$ in 2-dim momentum space.

➢A number of papers [1] have attempted to calculate $G(\mathbf{k};a,\varepsilon)$ using the multiple scattering formalism. They all found that

$$G(|\mathbf{k}|\to\infty;a,\varepsilon)\to|\mathbf{k}|\, g(a,\varepsilon) \quad \text{for all } a,\varepsilon(\omega)$$

➢This leads to **UV DIVERGENT** perturbative roughness corrections for perfectly acceptable roughness correlation functions such as:

$$\langle h(\mathbf{0})h(\mathbf{x})\rangle = \sigma^2 e^{-|\mathbf{x}|/l_c} \Rightarrow D(\mathbf{k}) = \frac{2\pi\sigma^2 l_c^{\,2}}{(\mathbf{k}^2 l_c^{\,2}+1)^{3/2}} \xrightarrow{|\mathbf{k}|\to\infty} \frac{2\pi\sigma^2}{l_c|\mathbf{k}|^3}$$

$$\text{with } \sigma^2 = \int D(\mathbf{k}) d\mathbf{k} \quad \text{but} \quad \int |\mathbf{k}|\, D(\mathbf{k}) d\mathbf{k} \to \infty$$

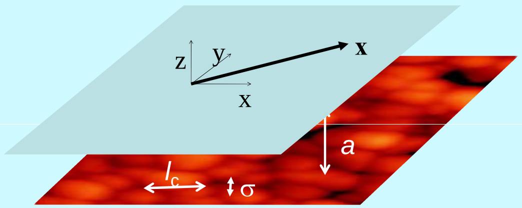
An asymptotic behavior of the loop function $G(|\mathbf{k}|\to\infty;a,\varepsilon)\sim|\mathbf{k}|$ thus leads to an UV-divergence that is physically unacceptable.

➢Previous treatments of roughness corrections ignored the fact that roughness coupling to surface plasmons gives superficially divergent corrections to the reflection coefficients at 1-loop. This correction has to be renormalized whether or not another plate is present.

[1]: Novikov,Sorin&Chernyak(1992); Buscher&Emig (2004); Neto, Lambrecht&Reynaud(2005)

The model

➢The whole two-plate roughness system is described by an EM, $\mathbf{E}_\xi(\mathbf{x},z)$ field living in 4D Euclidean space-time and a height field, $h(\mathbf{x})$ constrained to the surface of the rough plate.



Geometry for the Casimir interaction between a flat and a stochastically rough plate at a mean separation "a". The roughness is characterized by the variance σ^2 and correlation length l_c.

The interaction is described by the action:

$$S_{\text{int}} = \frac{i}{2}\int \frac{dz d\xi}{2\pi} \mathbf{E}_\xi(\mathbf{x},z) V_{\text{int}} \mathbf{E}_\xi(\mathbf{x},z) + i\int d\mathbf{x} h(\mathbf{x}) c_1(a) + \frac{i}{2}\int d\mathbf{y} d\mathbf{x} h(\mathbf{x}) c_2(\mathbf{x}-\mathbf{y}) h(\mathbf{y})$$

$$V_{\text{int}} = (\varepsilon(\xi)-1)[\theta(z-h(\mathbf{x}))-\theta(z)] + \delta(z) C(\xi)$$

Roughness is modeled by the generating function :

$$Z_h[\alpha] = e^{\frac{1}{2}\iint \alpha(\mathbf{x}) D(\mathbf{x}-\mathbf{y}) \alpha(\mathbf{y})} \quad \text{with } D(\mathbf{x}-\mathbf{y}) = \langle h(\mathbf{x}) h(\mathbf{y})\rangle := \frac{1}{A}\int_A d\mathbf{r}\, h(\mathbf{x}+\mathbf{r}) h(\mathbf{y}+\mathbf{r})$$

The total generating function for the model is :

$$Z[\alpha,\mathbf{j}] = \exp[-S_{\text{int}}(\frac{\delta}{\delta\mathbf{j}},\frac{\delta}{\delta\alpha})] \times Z_h[\alpha] \exp[\frac{1}{2}\iint \mathbf{j}\Gamma\mathbf{j}]$$

The Greens-dyadic, $\Gamma(\mathbf{x}-\mathbf{y},z,z',\xi)$ of two flat parallel plates satisfies (Schwinger):

$$[\mathbf{1}+\frac{1}{\xi^2}\nabla\times\nabla\times+(\boldsymbol{\varepsilon}-\mathbf{1})(\theta(z)+\theta(-z-a))]\Gamma(\mathbf{x}-\mathbf{y},z,z',\xi) = \mathbf{1}\delta(z-z')\delta(\mathbf{x}-\mathbf{y})$$

All components of the3X3 dyadic $\Gamma(\mathbf{x}-\mathbf{y},z,z',\xi)$ decompose as:

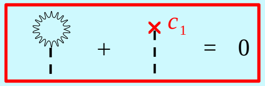
$$\Gamma_{xx'}\big|_{z=z'=0} = \Gamma^\infty_{xx'} + \Gamma^a_{xx'} = \frac{\kappa_\varepsilon\kappa}{\kappa_\varepsilon+\varepsilon\kappa} + \frac{2r_H\kappa\kappa_\varepsilon^2 e^{-2a\kappa}}{(\kappa_\varepsilon+\varepsilon\kappa)^2[1-r_H^2 e^{-2a\kappa}]} \Rightarrow \text{WWW}$$

$$r_E = \frac{\kappa-\kappa_\varepsilon}{\kappa+\kappa_\varepsilon} \qquad r_H = \frac{\varepsilon\kappa-\kappa_\varepsilon}{\varepsilon\kappa+\kappa_\varepsilon} \qquad \text{and} \qquad \kappa=\sqrt{k^2+\xi^2} \qquad \kappa_\varepsilon=\sqrt{k^2+\varepsilon\xi^2}$$

Feynman Diagrams – Counter terms

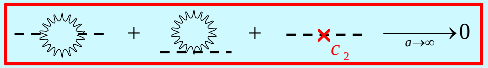
➢The one-point counter term, c_1, depends on the plate separation "a". It is introduced to ensure $\langle h(\mathbf{x})\rangle = 0$ when the interaction is turned on.

$$\left[\frac{\partial Z}{\partial\alpha}\right]_{\alpha=0} = \langle h(\mathbf{x})\rangle = 0 \quad \Rightarrow$$

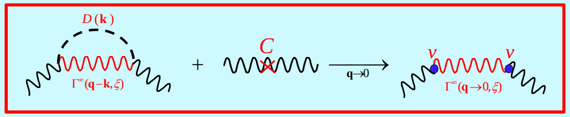


➢The two-point counter term, c_2, guarantees that the measured correlation of the profile $D(\mathbf{x}-\mathbf{y})$ when the two plates are far apart.

$$\left[\frac{\partial}{\partial\alpha}\frac{\partial}{\partial\alpha'}Z\right]_{\alpha=0,a\to\infty} = \langle h(\mathbf{x})h(\mathbf{y})\rangle = D(\mathbf{x}-\mathbf{y})$$

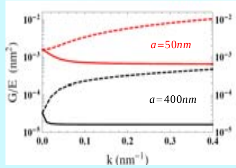


➢The two-point counter term, C, ensures that the coupling of long-wavelength EM excitations to the surface plasmon due to roughness is reproduced.



Determine coupling constant v by demanding PFA limit !

➢Instead of the linear asymptotic behavior of the response function $\mathbf{G}(\mathbf{k})$ of all previous approaches, the renormalized field theory gives a $\mathbf{G}(\mathbf{k})$ that approaches a finite value at high momenta.



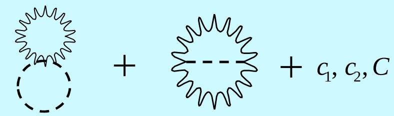
Two-Loops Corrections

➢The perturbative roughness corrections is the sum of two-loop Feynman diagrams of 1st order and 2nd order in the EM-dyadic (and one-loop diagrams that include the 1-loop counterterms).

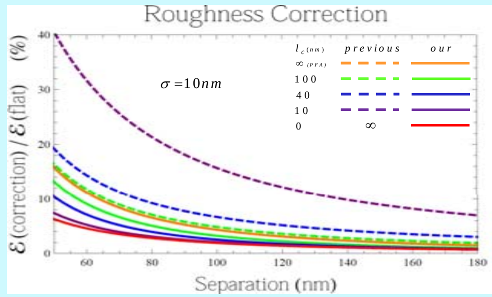
➢The 1st kind of diagram is local and enhances the force due to the change in average separation from the roughness profile.

➢The 2nd non-local correction, describes the scattering effect of roughness correlations and always decreases the attractive force.

$$\Delta\mathcal{E}_{Rough} =$$



➢Contrary to previous approaches, the corrections decrease as the roughness increases($l_c \to 0$) for any given profile variance.



Summary

➢We developed a field theoretical description of roughness corrections that allows their computation for any physically realizable roughness correlation function -- and is not restricted to Gaussian correlations.
➢As a bonus we determine the correct perturbative coupling of EM radiation to plasmons by roughness .
➢We found upper(PFA) and lower bounds for the roughness correction to Casimir energies. This is important for the interpretation of precision Casimir experiments.