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Electromagnetic Lifshitz formula for small-width mirrors

María Laura Remaggi (in collaboration with C. D. Fosco)

Instituto Balseiro, Universidad Nacional de Cuyo - Centro Atómico Bariloche - Comisión Nacional de Energía Atómica

Bariloche, Argentina

Instituto Balseiro Bariloche

Abstract

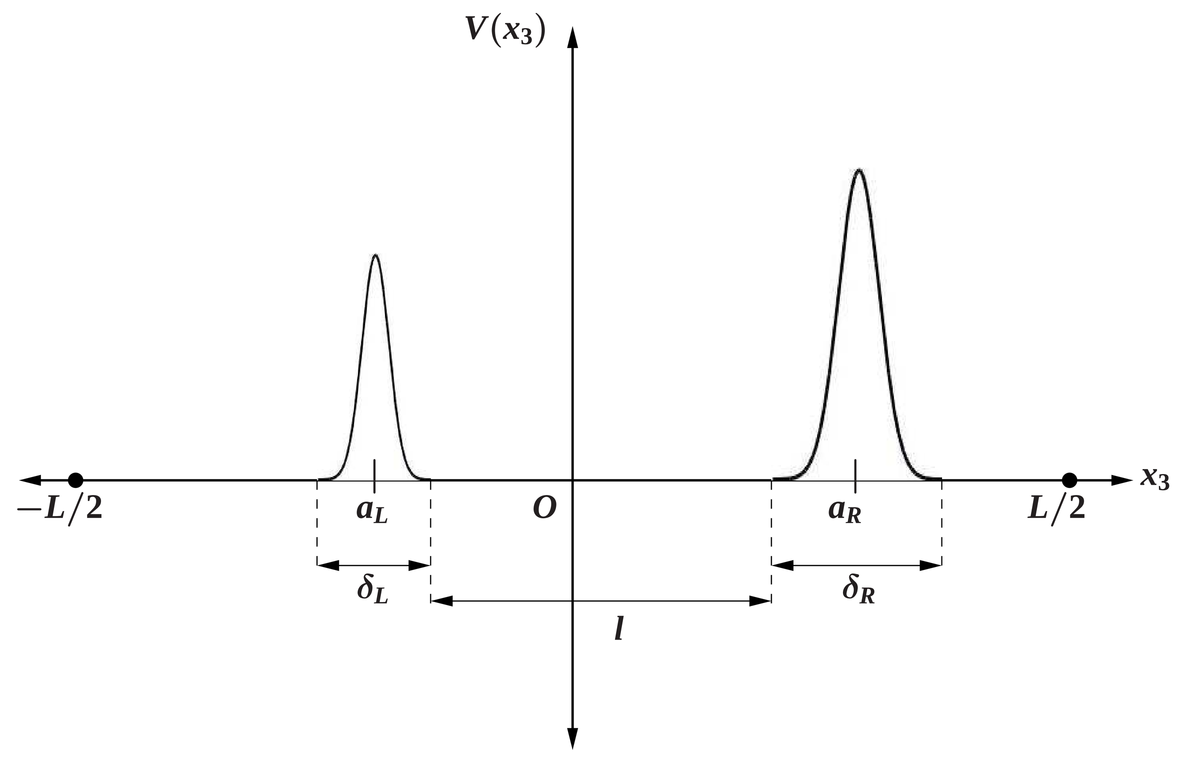
We present the results of our last work [1], where we extend a recently proposed Quantum Field Theory (QFT) approach to the Lifshitz formula, originally implemented for a real scalar field, to the case of a fluctuating vacuum electromagnetic (EM) field, coupled to two flat, parallel mirrors. The general result is presented in terms of the vacuum polarization tensors due to the media on each mirror. We consider small width mirrors, with the zero-width limit as a particular case. We apply the latter to models involving graphene sheets, obtaining results which are consistent with previous ones.

Motivations

- Lifshitz formula (LF) provides a useful tool for evaluating the Casimir force between mirrors.
- One important generalization of the LF consists in considering models with background potentials.
- In [2] a QFT approach was used to derive the LF for a fluctuating real scalar field coupled to two mirrors, represented by background potentials. The derivation relied upon the application of the Gelfand-Yaglom formula for functional determinants.
- The aim of this work is to adapt this approach to the case of a fluctuating Abelian gauge field.

The model

EM field + two mirrors modeled by background potentials



Euclidean action of the system

$$\mathcal{S}(A) \;=\; \mathcal{S}_0(A) \;+\; \mathcal{S}_{int}(A)$$

Free gauge field term

$$\mathcal{S}_0(A) \;=\; \int d^4x \left[\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2a}(\partial\cdot A)^2\right]$$

(we adopt $a \equiv 1$)

Interaction term

$$\mathcal{S}_{int} \;=\; \mathcal{S}_L \;+\; \mathcal{S}_R$$

$$\mathcal{S}_I(A) \;=\; \frac{1}{2}\int_{x_\parallel, x'_\parallel, x_3} A_\alpha(x_\parallel, x_3)\Pi^{(I)}_{\alpha\beta}(x_\parallel - x'_\parallel; x_3 - a_I)A_\beta(x'_\parallel, x_3)$$

$\Pi_{\mu\nu}$: vacuum polarization tensor
$x_\parallel \equiv (x_0, x_1, x_2)$
$I = L, R$
$\alpha, \beta = 0, 1, 2$ (small width$\equiv$ no current along x_3)

Formulation of the problem

We are interested in deriving an expression for the Casimir free energy per unit area:

Casimir free energy per unit area

$$\Gamma_C(\beta) \;=\; -\frac{1}{\beta}\lim_{L\to\infty}\left[\frac{1}{L^2}\log\frac{\mathcal{Z}(\beta)}{\mathcal{Z}_0(\beta)}\right]$$

Partition function

$$\mathcal{Z}(\beta) \;=\; \int \mathcal{D}A\, e^{-\mathcal{S}(A)}$$

$\mathcal{Z}_0 = \lim_{l\to\infty}\mathcal{Z}$. We discard l independent factors by taking $\frac{\mathcal{Z}(\beta)}{\mathcal{Z}_0(\beta)}$.

Relevant physical observables:

Vacuum energy per unit area

$$\mathcal{E}_{vac} = \lim_{\beta\to\infty}\Gamma_C(\beta)$$

Casimir force per unit area

$$\mathcal{F}_C(\beta) \;=\; -\frac{\partial\Gamma_C(\beta)}{\partial l}$$

Zero-temperature limit $\mathcal{F}_C \equiv \lim_{\beta\to\infty}\mathcal{F}_C(\beta)$

- We will decompose the problem of evaluating Γ_C into two independent one-dimensional systems, each one corresponding to a single real scalar field.
- We will use the results obtained in [2] for each real scalar field.
- We will apply the results to particular cases involving zero width mirrors.

Reduction to scalar systems

- Using current conservation, we decompose $\widetilde{\Pi}^{(I)}_{\alpha\beta}$ in terms of scalar functions:

Vacuum polarization tensor expansion

$$\widetilde{\Pi}^{(I)}_{\alpha\beta}(k_\parallel, x_3) \;=\; f_t^{(I)}(k_0^2, \mathbf{k}_\parallel^2, x_3)\,\mathcal{P}^t_{\alpha\beta} + f_l^{(I)}(k_0^2, \mathbf{k}_\parallel^2, x_3)\mathcal{P}^l_{\alpha\beta}$$

$$\mathcal{P}^t_{\alpha\beta} \equiv \breve{\delta}_{\alpha\beta} - \frac{\breve{k}_\alpha\breve{k}_\beta}{\breve{k}^2} \qquad \breve{k}_\alpha \equiv k_\alpha - k_0 n_\alpha$$

$$\mathcal{P}^l_{\alpha\beta} \equiv \mathcal{P}^\perp_{\alpha\beta} - \mathcal{P}^t_{\alpha\beta} \qquad \breve{\delta}_{\alpha\beta} \equiv \delta_{\alpha\beta} - n_\alpha n_\beta$$

$$\mathcal{P}^\perp_{\alpha\beta} \equiv \delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k_\parallel^2} \qquad n = (1,0,0) \qquad \mathbf{k}_\parallel \equiv (k_1, k_2)$$

- Performing Fourier transformation in $\mathcal{S}$ over the time and the parallel coordinates, and making use of the above result, we split $\mathcal{S}$ into two terms:

$$\mathcal{S} \;=\; \mathcal{S}_\parallel(\tilde{A}_\parallel) \;+\; \mathcal{S}_3(\tilde{A}_3)$$

- Decomposing $\tilde{A}_\parallel = \tilde{A}^{(t)} + \tilde{A}^{(l)}$, with $\tilde{A}^{(t,l)} \equiv \mathcal{P}^{(t,l)}\tilde{A}_\parallel$, the free energy becomes:

$$\Gamma_C(\beta) \;=\; \Gamma_t(\beta) + \Gamma_l(\beta)$$

$$\Gamma_{t,l}(\beta) \;=\; \frac{1}{2}\int_{k_\parallel}\log\left[\frac{\det \widetilde{T}_{t,l}(k_\parallel)}{\det \widetilde{T}_0(k_\parallel)}\right]$$

$$\widetilde{T}_{t,l}(k_\parallel) \;=\; -\partial_3^2 + k_\parallel^2 + \widetilde{V}_{t,l}(x_3, k_\parallel)$$

$$\widetilde{T}_0(k_\parallel) \;=\; -\partial_3^2 + k_\parallel^2$$

$$\widetilde{V}_{t,l}(x_3, k_\parallel) \;=\; \sum_I f^{(I)}_{t,l}(k_0^2, \mathbf{k}_\parallel^2, x_3 - a_I)$$

The system has been reduced to two independent Casimir problems, each one corresponding to a real scalar field in the presence of its background potential $\widetilde{V}_{t,l}$.

General result

- Applying the results of [2] to each scalar problem, we obtain:

$$\Gamma_{t,l}(\beta) \;=\; \frac{1}{2}\int_{k_\parallel}\log\left[1 + \frac{T_{12}^{(R)}T_{21}^{(R)}}{T_{11}^{(L)}T_{11}^{(L)}}e^{-2|k_\parallel|l}\right]_{t,l}$$

T is related to the solution of the following homogeneous equation

$$\widetilde{T}(k_\parallel)\psi(x_3) \;=\; 0$$

$$\Psi(x_f) \;=\; A(x_f, x_i)\,\Psi(x_i) \qquad T = B^{-1}AB$$

$$\Psi(x) = \begin{pmatrix}\psi(x)\\ \psi'(x)/\Omega\end{pmatrix} \qquad B = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

A particular case: Zero width mirrors

In this case:

$$f^{(I)}_{t,l}(k_0^2, \mathbf{k}_\parallel^2, x_3 - a_I) \;=\; \delta(x_3 - a_I)\, g^{(I)}_{t,l}(k_0^2, \mathbf{k}_\parallel^2)$$

Casimir force per unit area

$$\mathcal{F}_C(\beta) = \mathcal{F}_C^{(t)}(\beta) + \mathcal{F}_C^{(l)}(\beta)$$

$$\mathcal{F}_C^{(t,l)}(\beta) = -\int_{k_\parallel}\frac{|k_\parallel|g^{(L)}_{t,l}g^{(R)}_{t,l}e^{-2|k_\parallel|l}}{(2|k_\parallel| + g^{(L)}_{t,l})(2|k_\parallel| + g^{(R)}_{t,l}) - g^{(L)}_{t,l}g^{(R)}_{t,l}e^{-2|k_\parallel|l}}$$

For a graphene sheet:

$$g_t \;=\; \alpha\sqrt{k_0^2 + v_F^2\mathbf{k}_\parallel^2}$$

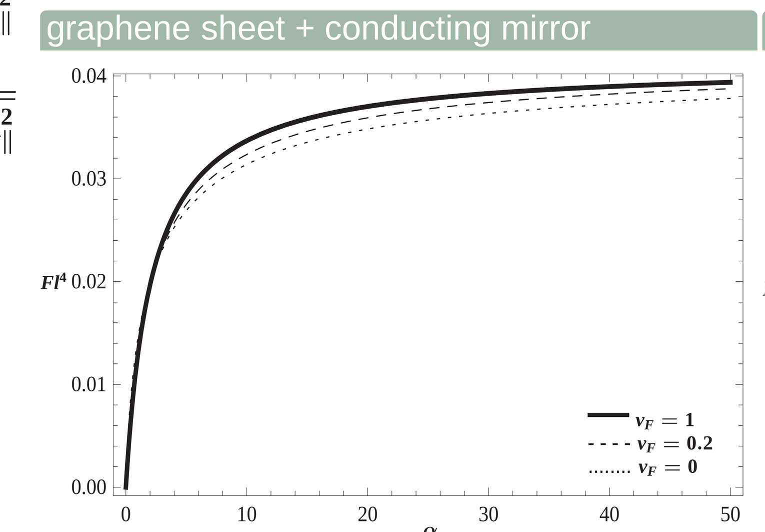
$$g_l \;=\; \alpha\frac{k_0^2 + \mathbf{k}^2}{\sqrt{k_0^2 + v_F^2\mathbf{k}_\parallel^2}}$$

$$\alpha \;=\; \frac{e^2N}{16}$$

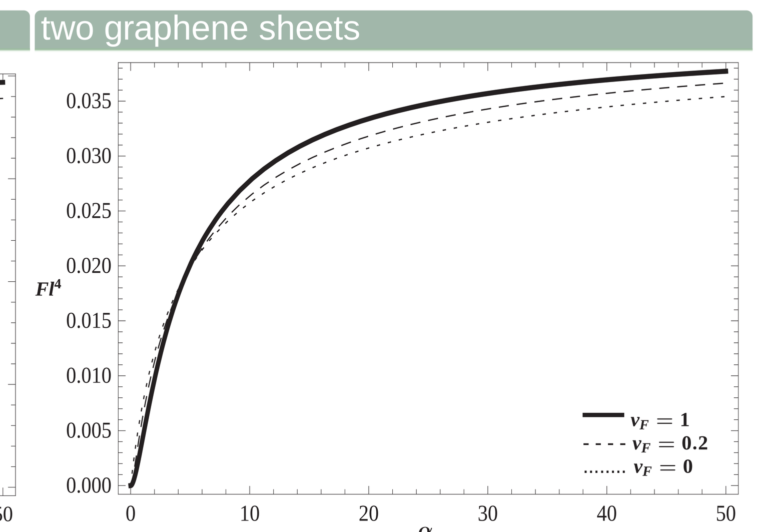
N: number of fermion flavours
e: coupling constant
v_F: Fermi velocity

conducting mirror:
$v_F = 1, \alpha\to\infty$

graphene sheet + conducting mirror



two graphene sheets



Conclusions

- We have derived a general expression for the Casimir free energy of a fluctuating Abelian gauge field in terms of the functional determinant.
- We have shown that, under some assumptions regarding the form of the coupling between the gauge field and the mirrors, the problem can be reduced to scalar systems, for which one can apply the previously known expression for the functional determinant.
- The result, expressed in terms of vacuum polarization tensors, enables to bypass the calculation of the reflection coefficients of each mirror, as it would be the case with the usual version of Lifshitz formula.
- The result allows to consider cases where the material media have a non trivial dependency along the normal direction.
- For zero width mirrors with graphene-like properties, we have shown that the QFT approach is consistent with the one in ([3],[4]).

References

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remaggi@cab.cnea.gov.ar