Relativistic effects in the dynamical Casimir effect in a superconducting circuit

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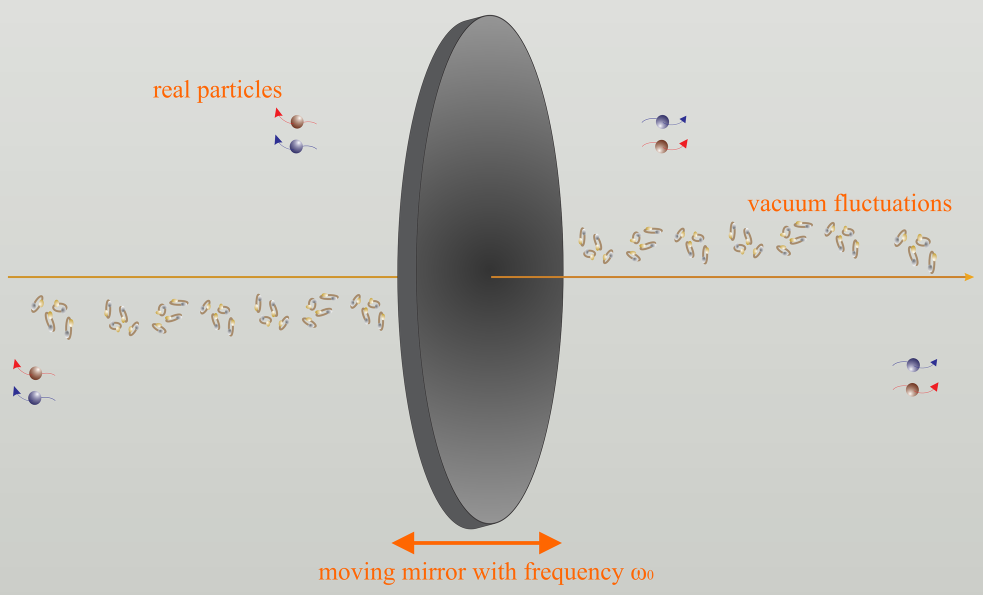
Introduction

- The **dynamical Casimir effect** (DCE) consists, basically, in:
  - particle creation due to moving mirrors;
  - radiation reaction forces on the moving mirrors.
- It already manifests for a single moving plate:



- 40 years after of the first theoretical paper (Moore 1970), the DCE was finally observed in the context of circuit-QED (Wilson *et al* 2011).

Experimental proposals

In addition, several experimental proposals were presented for the detection of the Casimir radiation. Briefly:

- MIR experiment (Braggio *et al* 2005);
- Superradiance experiment (Kim *et al* 2006);
- Superconducting circuit (J.R. Johansson *et al* 2009/2010);
- Parametric oscillator with a non-linear cristal slab inside (Dezael and Lambrecht 2010);
- Time modulation of the refractive index of the cavity filling medium (Faccio and Carusotto 2011);
- Excitation process of Rydberg atoms in rectangular (Kawacubo and Yamamoto 2011) and cylindrical (Naylor 2012) cavities with a non-stationary plasma mirror.

DCE in a superconducting circuit

- An open coplanar waveguide terminated by a SQUID, that is a very sensitive magnetometer (J.R. Johansson *et al* 2009/2010).
- The phase field operator $\Phi(t,x)$, described by a scalar massless Klein-Gordon equation in $1+1$ dimensions, keeps the electromagnetic field in the transmission line.
- The effective inductance of the SQUID can be tuned by an external time-dependent magnetic flux, $\Phi_{ext}(t)$, providing a tunable BC (the second derivative in time was disregarded),

$$\Phi(t,0) + \Phi_0^2\left[(2\pi)^2 E_j(t) L_0\right]^{-1}\partial_x\Phi(t,0) \;=\; 0, \tag{1}$$

where $E_j(t)$ is the magnetic flux-dependent effective Josephson energy of the SQUID and L_0 is the characteristic inductance of the coplanar waveguide.

- This set-up is equivalent to a one-dimensional transmission line with a tunable effective length, i.e., a moving boundary.
- The effective velocity is around $0.1c$, leading to high photon creation rates.

Relativistic effects in the dynamical Casimir effect

- The presence of additional bands was already reported in literature for a Dirichlet condition and mechanical oscillation of a single mirror (Lambrecht *et al* (1998)).
- In that context, the additional sidebands were considered "not realistic as it would imply a mirror's mechanical velocity appreciable compared to the speed of light".
- **Purpose:** to study relativistic effects in the particle creation, particularly, the possibility of the appearance of an additional band in the spectral density considering the theoretical model underlying the SQUID experiment.

- Massless scalar field in $1+1$ dimensions that satisfies the Klein-Gordon equation $(\partial_t^2 - \partial_x^2)\phi\,(t,x) = 0$ in the presence of a static mirror at $x = 0$ and obeying Robin BC with a time-dependent parameter $\gamma(t)$ (Silva *et al* 2011):

$$\phi\,(t,0) = \gamma\,(t)\,\partial_x\phi\,(t,0)\,, \tag{2}$$

where we consider $\gamma(t) = \gamma_0 + \epsilon F\,(t)$, with ϵ a non-dimensional constant. Be aware that γ_0 is the usual Robin parameter and $F\,(t)$ describes its deviation in time.

- Comparing the tunable BC (1) with Eq. (2), we can map

$$\gamma(t) = -\Phi_0^2\left[(2\pi)^2 E_j(t) L_0\right]^{-1} = -L_{\text{eff}}\,(t)\,, \tag{3}$$

where Φ_0 is the magnetic flux quantum and $L_{\text{eff}}\,(t)$ is an effective length that modulates the change in time of the distance between the SQUID to an effective mirror at origin (J.R. Johansson *et al* 2009/2010).

- Considering the following general expression for the Josephson energy, $E_j\,(t) = E_j^0 + \delta E_j\,(t)$, that implies in $L_{\text{eff}}(t) = L_{\text{eff}}^0 + \delta L_{\text{eff}}(t)$, we can identify from $\gamma(t) = \gamma_0 + \epsilon F\,(t)$:

$$\gamma_0 = -L_{\text{eff}}^0, \quad \epsilon F(t) = -\delta L_{\text{eff}}(t), \tag{4}$$

that establish a reasonable link between the DCE in superconducting circuits and time-dependent Robin BC.

- We have plausible arguments to ensure that Eq. (1) is closer to a Robin BC with time-dependent parameter than a Dirichlet one:
  1. The reflection coefficient of the phase field operator for a static magnetic flux is totally similar to the usual Robin BC case (Mintz *et al* 2006).
  2. The relevant frequencies in the superconducting circuit experiment are much smaller than the plasma frequency of the SQUID that is another nice property of the Robin BC (Mintz *et al* 2006).
- In the Ford-Vilenkin perturbative approach (Ford and Vilenkin 1982), the field operator $\phi\,(t,x)$ is expressed by

$$\phi\,(t,x) \approx \phi_0\,(t,x) + \epsilon\phi_1\,(t,x)\,, \tag{5}$$

where $\phi_0\,(t,x)$ is the unperturbed field and $\phi_1\,(t,x)$ represents the first order perturbation in the parameter ϵ.

- In the relativistic regime, it is natural including terms higher than the first order perturbation. For this reason, we consider a generalization of the Ford-Vilenkin approach

$$\phi\,(t,x) \approx \phi_0\,(t,x) + \sum_{j=1}^{N}\epsilon^j\phi_j\,(t,x)\,, \tag{6}$$

where N denotes the order of expansion under investigation. For each order added, the perturbed field ϕ_j satisfies the Klein-Gordon equation $(\partial_t^2 - \partial_x^2)\phi_j\,(t,x) = 0$ with the following general BC:

$$\phi_j\,(t,0) - \gamma_0\partial_x\phi_j\,(t,0) = F(t)\,\partial_x\phi_{j-1}\,(t,0)\,, \tag{7}$$

where terms at order ϵ^{j+1} are suppressed.

- The spectral distribution of created particles between ω and $\omega + d\omega$ in $1+1$ dimensions is given by:

$$dN(\omega)/d\omega = \left\langle \left(\hat{a}_\omega^{out}\right)^\dagger \hat{a}_\omega^{out}\right\rangle. \tag{8}$$

- From the Bogoliubov transformations that be computed with the help of Fourier transform of Eqs. (6) and (7) it is straightforward to show that

$$\begin{aligned}\frac{dN(\omega)}{d\omega} = 4\omega\sum_{j=1}^{N}\sum_{k=1}^{N}\epsilon^{j+k}\int_{-\infty}^{\infty} d\xi\,\Theta(-\xi)\left[\xi\left(1+\xi^2\gamma_0^2\right)\right]^{-1} \\ \times\left\{\mathcal{O}_{\omega,\xi}^{(j)*}\,(x)\left[\sin\left(\xi x\right) + \xi\gamma_0\cos\left(\xi x\right)\right]\right\}_{x=0} \\ \times\left\{\mathcal{O}_{\omega,\xi}^{(k)}\,(x)\left[\sin\left(\xi x\right) + \xi\gamma_0\cos\left(\xi x\right)\right]\right\}_{x=0},\end{aligned} \tag{9}$$

which predicts the occurrence of additional bands in the particle creation spectrum for a particular choice of $F\,(t)$. Notice that

$$\begin{aligned}\mathcal{O}_{\omega,\xi}^{(j)}\,(x) &= \int_{-\infty}^{\infty}\frac{d\xi_1}{2\pi}\Gamma\left(\omega - \xi_1\right)\partial_x\mathcal{O}_{\xi_1,\xi}^{(j-1)}\,(x)\,,\\ \mathcal{O}_{\xi_1,\xi}^{(0)}\,(x) &= \delta\left(\xi_1 - \xi\right)\,,\end{aligned} \tag{10}$$

represent recurrence formulas for the spatial operators $\mathcal{O}$ that act on the Fourier transform of the unperturbed field. The function $\Gamma(\omega)$ is the Fourier transform of $F(t)$.

- As an illustration, considering $N = 2$ and for a typical oscillatory time variation of the Robin parameter, namely, $F(t) = \epsilon_0\cos\left(\omega_0 t\right)e^{-|t|/\tau}$, where ϵ_0 is the amplitude, ω_0 is the characteristic frequency and τ is the effective time interval of the oscillation, we have found analytical formulas for the particle creation spectrum: $dN(\omega)/d\omega = \epsilon^2\mathcal{N}_2(\omega) + \epsilon^4\mathcal{N}_4(\omega)$, where $\mathcal{N}_2(\omega)$ immediately recover the non-relativistic particle spectral distribution (Silva *et al* 2011) and $\mathcal{N}_4(\omega)$ represents the additional band at the referred order.

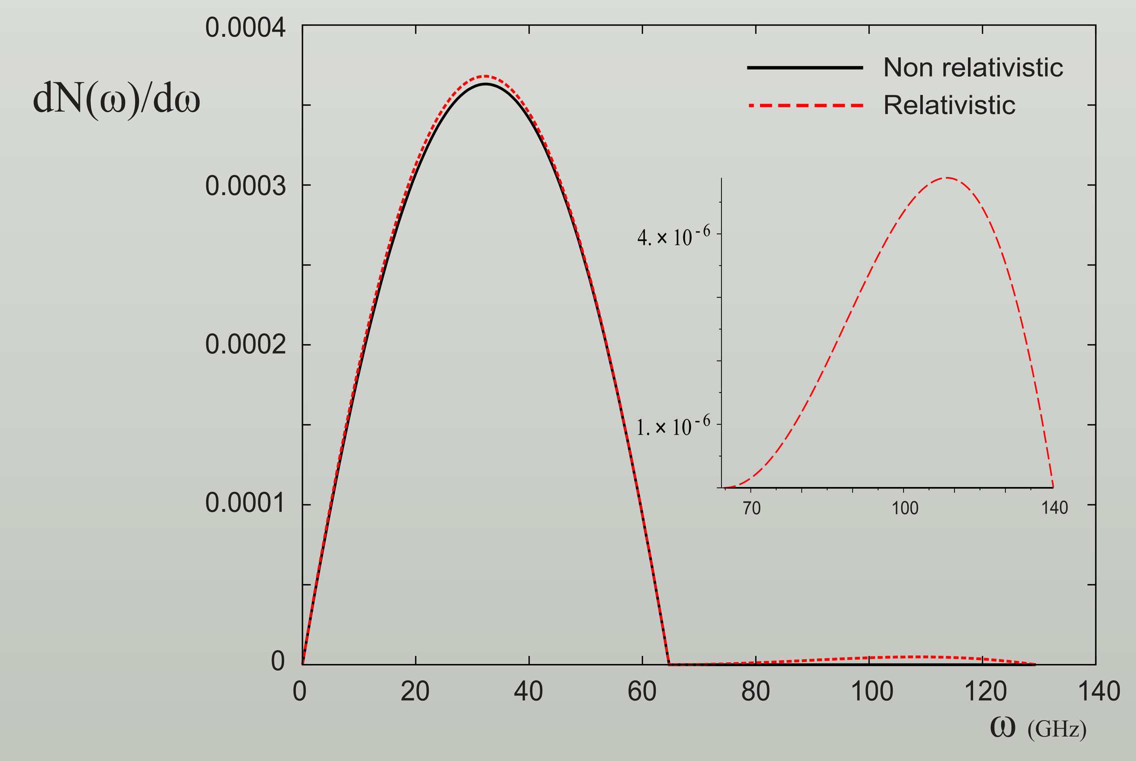


Figure 1: Particle creation spectrum for $\omega_0 = 2\pi\times 10,30$GHz, $\epsilon_0 = 0,46\times 10^{-3}$m, $v_e = 0,1c$ and $\epsilon_0/\gamma_0 = 0,25$. Note the presence of a second band in the spectrum, where the ratio between second and first bands is around 1%. The constants $\hbar$ and c are recovered to ensure the correct dimension of the particle creation spectrum

- In Fig. 1, we notice the rising of a second band of created particles in the spectral distribution when the velocity of the effective mirror can achieved 10% of the speed of light. The inset highlight the assymetric shape of the second band. The ratio between the peak of the second and first bands is around 0.1.
- Finally, thermal effects can be computed. Apart from the vacuum contribution, two others terms contribute to the spectral distribution. The first one is purely classical and corresponds to the particles already presented in the thermal bath. The second one represents the amplification of the particles immersed in the thermal bath and its functional structure is similar of Eq. (9), substituting the Heaviside function by the thermal occupation number $\bar{n}_T\,(\xi) = \left[e^{|\xi|/T} - 1\right]^{-1}$.

Final remarks

In summary, we investigated relativistic effects for the spectral distribution of created particles in the experimental model that observed the DCE. The existence of an additional band in the particle created spectrum is a natural consequence due to relativistic effects. We estimate that the peak of the second band occurs approximately at $(1+0.1)\,3\omega_0/2$, revealing the asymmetry on the second band in contrast to the first one. The thermal spectral distribution for the second band can be disregarding which confirms that the relativistic effects are governed by quantum vacuum fluctuations. The observation of these relativistic effects could be of some experimental interest and represents another signature of the DCE.

Acknowledgments

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