Relativistic effects in the dynamical Casimir effect in a superconducting circuit
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Introduction

- The dynamical Casimir effect (DCE) consists, basically, in:
  - particle creation due to moving mirrors;
  - radiation reaction forces on the moving mirrors.
- It already manifests for a single moving plane.

- Massless scalar field in 1 + 1 dimensions that satisfies the Klein-Gordon equation \( \mathcal{L}_\text{eff}(t, x) = 0 \) in the presence of a static mirror at \( x = 0 \) and obeying Robin BC with a time-dependent parameter \( \gamma(t) \) (Silva et al. 2011):
  \[ \phi(t, 0) = \gamma(t) \theta(x, t), \quad \text{with} \quad \gamma(t) = \gamma_0 + \epsilon \tau(t), \quad \text{where} \quad \epsilon \text{ is a non-dimensional constant.} \]
  Be aware that \( \gamma(t) \) is the usual parameter and \( F(t) \) describes its deviation in time.

- Comparing the tunable BC (1) with Eq. (2), we can map
  \[ \gamma(t) = -\Omega_0^2 \left( 2\pi \right)^2 E_i(t) L_0^{-1} = -\frac{1}{L_{\text{eff}}}(t), \]
  \( \text{where} \quad L_{\text{eff}} = \frac{m^2}{\Delta E_{\text{eff}}} = L + \delta L_{\text{eff}}(t) = \text{an effective length that modifies the change in time of the distance between the SQUID and an effective mirror at origin (J.R. Johansson et al. 2009/2010).} \]

- Taking into account the generalisation of the Klein-Gordon equation \( \Box \phi - m^2 \phi = 0 \) (Dhruw R. Lefebvre, 2011), we can identify from \( \gamma(t) = \gamma_0 + \epsilon T(t) \),
  \[ \gamma = -\Omega_0^2 \left( 2\pi \right)^2 E_i(t) L_0^{-1} = -\frac{1}{L_{\text{eff}}}(t), \]
  \( \text{that establish a reasonable link between the DCE in superconducting circuits and time-dependent Robin BC.} \)

- We have plausible arguments to ensure that Eq. (1) is closer to a Robin BC with time-dependent parameter \( \text{a Drichlet one:} \)
  \( \phi(t, x) \approx \phi_0(t, x) + \epsilon \delta(t, x) \),
  \( \gamma(t) \text{ is the unperturbed field and } \phi_0(t, x) \text{ represents the first order perturbation in the parameter } \epsilon. \)

- In the relativistic regime, it is natural including terms similar to the first order perturbation. For this reason, we consider a generalization of the Robin-Vilenkin approach
  \[ \phi(t, x) \approx \phi_0(t, x) + \sum_n \epsilon_n \phi_n(t, x) \]
  \( \text{where } \epsilon_N \text{ denotes the order of expansion under investigation. For each order added, the perturbed field } \phi_i \text{satisfies the Klein-Gordon equation } \Box \phi_i - m^2 \phi_i = 0 \text{ with the following general BC:} \)
  \[ \phi_i(t, x) \approx \phi_0(t, x) + \epsilon \delta(t, x) \phi(t, x) \]
  \( \epsilon \text{ where terms of } \epsilon^N \text{ are suppressed.} \)

- The spectral distribution of created particles between \( \omega \) and \( \omega + d\omega \) in 1 + 1 dimensions is given by:
  \[ dN(\omega)/d\omega = \left( \frac{\omega}{\omega_0^\alpha} \right)^\beta \frac{d\omega}{\omega} \]
  \( \text{where } \omega_0 \text{ is the cutoff frequency, } \alpha \text{ and } \beta \text{ are the spectral indices.} \)

- From the Bogoliubov transformations that be computed with the help of Fourier transform of Eqs. (6) and (7) it is straightforward to show that
  \[ dN(\omega)/d\omega = \sum_{n, -n} \frac{1}{\beta} \frac{\omega^{\alpha+n}}{\omega_0^\alpha} \left( \cos \xi_0 - \cos \xi_n \right) \left( \cos \xi_0 - \cos \xi_n \right)^{-1} \]
  \[ \times \left( \sin \xi_n \right) \left( \sin \xi_n \right)^{-1} \left( \cos \xi_0 + \cos \xi_n \right)^{-1} \left( \sin \xi_0 \right)^{-1} \left( \sin \xi_0 \right)^{-1} \left( \cos \xi_0 - \cos \xi_n \right)^{-1} \]
  \( \text{which predicts the occurrence of additional bands in the particle creation spectrum for a particular choice of } F(t) \text{. Notice that} \)
  \[ \mathcal{O} \mathcal{O}^{-1} = \mathcal{O} \mathcal{O}^{-1} \] (10)
  \( \text{represent recurrence formulas for the spatial operators } \mathcal{O} \text{ that act on the Fourier transform of the unperturbed field. The function } \Gamma(\omega) \text{ is the Fourier transform of } F(t). \)

Final remarks

- In summary, we investigated relativistic effects for the spectral distribution of created particles in the experimental model that observed the DCE. The existence of an additional band in the particle created spectrum is a natural consequence due to relativistic effects.
- We estimate that the peak of the second band occurs approximately at \( (1 + 0.1) \omega_0/2 \), revealing the asymmetry on the second band in contrast to the first one. The thermal spectral distribution for the second band can be disregarding which confirms that the relativistic effects are governed by quantum vacuum fluctuations. The observation of these relativistic effects could be of some experimental interest and represents another signature of the DCE.

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References