

Constraints on non-Newtonian gravity and light elementary particles from measurements of the Casimir force by means of dynamic AFM

G. L. Klimchitskaya,¹ U. Mohideen,² and V. M. Mostepanenko¹

¹Central Astronomical Observatory at Pulkovo of the Russian Academy of Sciences, St.Petersburg, 196140, Russia

and

Institute for Theoretical Physics, Leipzig University, Postfach 100920, D-04009, Leipzig, Germany

²Department of Physics and Astronomy, University of California, Riverside, California 92521, USA

Abstract

We derive constraints on corrections to Newtonian gravity of the Yukawa type and light elementary particles from two recently performed measurements of the gradient of the Casimir force. In the first measurement the configuration of two Au surfaces has been used, whereas in the second a nonmagnetic metal Au interacted with a magnetic metal Ni. In these configurations one arrives at different, respectively, similar theoretical predictions for the Casimir force when the competing theoretical approaches are employed. Nevertheless, we demonstrate that the constraints following from both experiments are in mutual agreement and in line with constraints obtained from earlier measurements. This confirms the reliability of constraints on non-Newtonian gravity obtained from measurements of the Casimir force.

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I. INTRODUCTION

Deviations from Newton's gravitational law are presently discussed in many different aspects in connection with various extensions of the Standard Model and the problem of dark matter. The Yukawa-type corrections to Newtonian gravity in the wide interaction range from nanometers to 10^{15} cm are predicted [1, 2] by the extra-dimensional models with a low-energy compactification scale of the order of 1 TeV [3–5]. The modified Newtonian dynamics is discussed [6] as an alternative to the proposed existence of dark matter which should comprise about 23% of the Universe mass. On the other hand, if it is granted that the most realistic according to astrophysics, cold, dark matter exists in nature, the question arises as to what are its constituents. One of the answers to this question is that the cold dark matter consists of non-thermally produced axions [7]. The exchange of such type of particles between atoms of two macrobodies generates an effective Yukawa-type correction to Newton's gravitational law with an interaction range λ varying from 1 \AA to hundreds of meters or even larger depending on the mass of a particle $m = \hbar/(\lambda c)$.

Constraints on corrections to Newtonian gravity are under investigation for many years [8]. Strong constraints within the interaction range from millimeters to meters were obtained from the Cavendish- and Eötvös-type experiments [9, 10]. In the submillimeter interaction range stronger constraints on the Yukawa-type corrections were currently derived from Cavendish-type experiments [11–13]. The strength of these constraints, however, decreases with decreasing λ . For λ smaller than a few micrometers the strongest constraints on the Yukawa-type corrections to Newtonian gravity follow from measurements of the Casimir force [14, 15] which becomes the dominant background force in place of gravitation at sufficiently short separations between the test bodies (see review [16]). Measurements of the lateral Casimir force between sinusoidally corrugated surfaces of a sphere and a plate by means of an atomic force microscope (AFM) [17, 18] and the gradient of the Casimir force between smooth surfaces of a sphere and a plate by means of a micromachined oscillator [19, 20] have already resulted [21] in up to several orders of magnitude stronger constraints.

Constraints on the parameters of non-Newtonian gravity are commonly obtained from the measure of agreement between the experimental data for the Casimir force and theoretical predictions based on the Lifshitz theory [14]. It is necessary to stress, however, that theory-experiment comparison in the field of the Casimir effect has led to unexpected results

which are yet to be fully explained. Specifically, it was found that for metallic test bodies the theoretical predictions are in agreement with the experimental data if the relaxation properties of conduction electrons are omitted [14, 15, 19, 20, 22–25]. An experiment [26], showing agreement between the data and theory with the relaxation properties included, measured the sum of the Casimir force and up to an order of magnitude larger residual electric force supposedly originating from large patches. It was not possible to independently measure this residual electric force. Because of this, the model description for it has been used containing two fitting parameters. The values of fitting parameters were determined from the fit between the experimental data and theory. As was pointed out in the literature [27, 28], at short separations Ref. [26] neglects the role of surface imperfections of a spherical lens with centimeter-size radius of curvature. It was also shown [29, 30] that even with account of patch potentials, at large separations the experimental data of Ref. [26] for the total force are in better agreement with theory neglecting the contribution of charge carriers. Note that the issue concerning the role of patch potentials has yet not been finally solved. The experimental data of Refs. [22–25] are consistent with theory of Ref. [31], predicting negligibly small contribution of patches in the configurations of these experiments. For an alternative theory of patches see Ref. [32] and its discussion in Ref. [24].

For dielectric test bodies the theoretical predictions were found in agreement with the data only if the contribution of free charge carriers is omitted [33–38]. Keeping in mind that many of the experiments mentioned above were used, first, to make a selection between different theoretical approaches and, second, to constrain corrections to Newtonian gravity from the measure of agreement between the data and the predictions following from a selected approach, the constraints obtained were sometimes claimed to be of dubious merit [39]. This claim is questionable [40] because the difference between the excluded and confirmed theoretical approaches to the Casimir force cannot be modeled by the correction to Newtonian gravity of Yukawa type. It would be desirable, however, to have an independent confirmation of the previously obtained constraints that is not connected with a selection between different competing models of the Casimir force.

In this paper we derive constraints on non-Newtonian gravity and on the parameters of possible constituents of dark matter following from two recent measurements of the gradient of the Casimir force by means of dynamic AFM operated in the frequency-shift technique [24, 25, 41]. The first of these experiments [24, 25] deals with a hollow, Au-coated glass sphere

oscillating in close proximity to an Au-coated sapphire plate. In some sense the experiment [24, 25] is similar to experiments [19, 20, 22, 23], but it is performed using quite different laboratory setup (an AFM instead of a micromachined oscillator). The experimental data were compared with different theoretical approaches to the Casimir force and again were found to be in favor of the approach with the relaxation properties of conduction electrons omitted [24, 25]. Here we obtain constraints on non-Newtonian gravity following from this experiment and demonstrate that they are in agreement with those obtained in Refs. [19, 20] (but slightly weaker in the same proportion as the ratio of experimental errors in both experiments).

In the second experiment considered in this paper the gradient of the Casimir force acting between an Au-coated hollow glass sphere and a Si plate coated with a ferromagnetic metal Ni was measured by means of a dynamic AFM [41]. This configuration is of outstanding interest for constraining corrections to Newtonian gravity because, as was shown in Refs. [42, 43], within the range of experimental separations one obtains almost coincident theoretical Casimir forces, irrespective of whether the relaxation properties are included or omitted. Thus, we arrive at constraints on the parameters of non-Newtonian gravity which are independent of a selection between different theoretical approaches to the Casimir force. These constraints turn out to be slightly stronger than those obtained from the AFM experiment with two Au surfaces, because the magnetic experiment extends to smaller separation distances. At the same time, the constraints obtained here from the magnetic experiment are slightly weaker than those obtained previously in Refs. [19, 20] due to relatively lower precision of AFM measurements, as compared with measurements performed by means of a micromachined oscillator. Thus, the strongest constraints on non-Newtonian gravity derived in Refs. [19, 20] receive further independent substantiation.

The paper is organized as follows. In Sec. II we obtain constraints on non-Newtonian gravity following from the dynamic AFM experiment with two Au surfaces. In Sec. III the same is done using the measurement data of the AFM experiment with ferromagnetic plate. Section IV contains our conclusions and discussion.

II. CONSTRAINTS FROM THE DYNAMIC AFM EXPERIMENT WITH TWO AU TEST BODIES

The experiment [24, 25] considered in this section is in some analogy to earlier performed experiments of Refs. [19, 20, 22, 23], but it uses a dynamic AFM instead of a micromechanical torsional oscillator as a measurement device. The gradient of the Casimir force was measured between an Au-coated hollow glass microsphere, attached to the cantilever of an AFM, and an Au-coated sapphire plate within the range of separations from 235 to 500 nm. The thickness and density of the glass spherical envelope were $\Delta^{(s)} = 5 \mu\text{m}$ and $\rho_s = 2.5 \times 10^3 \text{ kg/m}^3$. For technological purposes the glass sphere was coated first with a layer of Al having a thickness $\Delta_{\text{Al}}^{(s)} = 20 \text{ nm}$ and density $\rho_{\text{Al}} = 2.7 \times 10^3 \text{ kg/m}^3$ and then with a layer of Au having the thickness $\Delta_{\text{Au}}^{(s)} = 280 \text{ nm}$ and density $\rho_{\text{Au}} = 19.28 \times 10^3 \text{ kg/m}^3$. The external radius of a coated sphere was measured to be $R = 41.3 \mu\text{m}$ [24, 25]. The sapphire plate of density $\rho_p = 4.1 \times 10^3 \text{ kg/m}^3$, which can be considered as infinitely thick, was coated with an Al layer of thickness $\Delta_{\text{Al}}^{(p)} = \Delta_{\text{Al}}^{(s)}$ and with an external Au layer of thickness $\Delta_{\text{Au}}^{(p)} = \Delta_{\text{Au}}^{(s)}$.

The Casimir interaction between the sphere and the plate measured in the experiment [24, 25] coexists with Newtonian gravitation and possible corrections to it. For calculations of the Casimir force both test bodies can be considered as made of bulk Au [14]. The gravitational force and corrections to it should be calculated, however, taking into account the layer structure of the sphere and plate. We admit that the corrections to Newtonian gravitation has the Yukawa form, so that the additional interaction to the Casimir interaction between the two point-like masses m_1 and m_2 can be described by the potential

$$V(r) = V_N(r) + V_{\text{Yu}}(r) = -\frac{Gm_1m_2}{r} (1 + \alpha e^{-r/\lambda}). \quad (1)$$

Here, r is the separation distance between the masses, G is the Newtonian gravitational constant, α and λ are the strength and interaction range of the Yukawa interaction. It can be easily seen [44] that in the experimental configurations under consideration the Newtonian gravitational force is much smaller than the error in the measurement of the Casimir force and can be neglected. The total Yukawa interaction energy is obtained by the integration over the volumes of a sphere V_s and a plate V_p :

$$V_{\text{Yu}}(a) = -G\alpha \int_{V_p} d^3r_1 \rho_p(\mathbf{r}_1) \int_{V_s} d^3r_2 \rho_s(\mathbf{r}_2) \frac{e^{-|\mathbf{r}_1 - \mathbf{r}_2|/\lambda}}{|\mathbf{r}_1 - \mathbf{r}_2|}, \quad (2)$$

where $\rho_p(\mathbf{r}_1)$ and $\rho_s(\mathbf{r}_2)$ are the respective mass densities and a is the closest separation between the bodies. Then the Yukawa force and its gradient are given by

$$F_{\text{Yu}}(a) = -\frac{\partial V_{\text{Yu}}(a)}{\partial a}, \quad \frac{\partial F_{\text{Yu}}(a)}{\partial a} = -\frac{\partial^2 V_{\text{Yu}}(a)}{\partial a^2}. \quad (3)$$

After performing the integration in Eq. (2) with account of the layer structure of a sphere and a plate, one obtains from Eq. (3) [45]

$$\frac{\partial F_{\text{Yu}}(a)}{\partial a} = -4\pi^2 G\alpha\lambda^2 e^{-a/\lambda} X^{(s)}(\lambda)X^{(p)}(\lambda), \quad (4)$$

where the following notations are introduced:

$$\begin{aligned} X^{(p)}(\lambda) &= \rho_{\text{Au}} - (\rho_{\text{Au}} - \rho_{\text{Al}})e^{-\Delta_{\text{Au}}^{(p)}/\lambda} \\ &\quad - (\rho_{\text{Al}} - \rho_p)e^{-(\Delta_{\text{Au}}^{(p)} + \Delta_{\text{Al}}^{(p)})/\lambda}, \\ X^{(s)}(\lambda) &= \rho_{\text{Au}}\Phi(R, \lambda) - (\rho_{\text{Au}} - \rho_{\text{Al}})\Phi(R - \Delta_{\text{Au}}^{(s)}, \lambda)e^{-\Delta_{\text{Au}}^{(s)}/\lambda} \\ &\quad - (\rho_{\text{Al}} - \rho_s)\Phi(R - \Delta_{\text{Au}}^{(s)} - \Delta_{\text{Al}}^{(s)}, \lambda)e^{-(\Delta_{\text{Au}}^{(s)} + \Delta_{\text{Al}}^{(s)})/\lambda} \\ &\quad - \rho_s\Phi(R - \Delta_{\text{Au}}^{(s)} - \Delta_{\text{Al}}^{(s)} - \Delta^{(s)}, \lambda)e^{-(\Delta_{\text{Au}}^{(s)} + \Delta_{\text{Al}}^{(s)} + \Delta^{(s)})/\lambda}, \\ \Phi(r, \lambda) &= r - \lambda + (r + \lambda)e^{-2r/\lambda}. \end{aligned} \quad (5)$$

The experimental data for the gradient of the Casimir force were found [24, 25] to exclude the theoretical approach using the Lifshitz theory and the dielectric permittivity obtained from the tabulated optical data of Au extrapolated to zero-frequency by means of the Drude model. The same force data turned out to be consistent with theory when the simple plasma model is used for the extrapolation to zero frequency within the limits of experimental errors $\Delta_{F'}(a)$ in the measurement of the force gradient which were determined at the 67% confidence level [24, 25]. In the limits of these errors no interactions of Yukawa type were observed. Thus, the constraints on the parameters of Yukawa interaction α and λ can be obtained from the inequality

$$\left| \frac{\partial F_{\text{Yu}}(a)}{\partial a} \right| \leq \Delta_{F'}(a). \quad (6)$$

Equations (4) and (5) were substituted in inequality (6). It was shown that the strongest constraints follow at the shortest separation $a = 235$ nm. In the measurement scheme with applied compensating voltage $\Delta_{F'}(a) = 0.50$ $\mu\text{N/m}$ [24]. The resulting constraints are shown as line 1 in Fig. 1, where the region of (λ, α) above the line is prohibited and below the line is allowed by the results of this experiment. The interaction region from $\lambda_{\text{min}} = 20$ nm to

$\lambda_{\max} = 3 \mu\text{m}$ shown in Fig. 1 corresponds to the masses of a hypothetical particle (axion, for instance) in the region from 66 meV to 9.9 eV. This overlaps with the typical mass scale from $m_a = 1 \mu\text{eV}$ to $m_a = 1 \text{eV}$ allowed for an axion [46] and includes part of the region allowed by the cosmological bound for relic thermal axions $m_a < 0.42 \text{eV}$ [47], which are also considered in a cosmological context along with non-thermally produced axions. From the line 1 of Fig. 1 the allowed interaction strength at $\lambda = \lambda_{\min} = 20 \text{nm}$ is $\alpha < 5.9 \times 10^{18}$. With increasing λ (or, respectively, decreasing axion mass) the constraints shown by line 1 become stronger. At $\lambda = \lambda_{\max} = 3 \mu\text{m}$ one obtains $\alpha < 3.6 \times 10^{10}$. In the next section the constraints of line 1 in Fig. 1 are compared with other constraints following from measurements of the Casimir force.

In the end of this section we note that the constraints of line 1 in Fig. 1 were obtained using the exact expressions (4), (5) for the gradient of the Yukawa force between a sphere and a plate [45]. Nearly the same results can be obtained in a more simple way by calculating the gradient of the Yukawa force in the framework of the proximity force approximation (PFA)

$$\frac{\partial F_{\text{Yu}}(a)}{\partial a} = -2\pi R P_{\text{Yu}}(a), \quad (7)$$

where $P_{\text{Yu}}(a)$ is the Yukawa pressure between two parallel plates having the same layer structure as a plate and a sphere in the experiment under consideration. The approximate Eq. (7) is applicable under the conditions

$$\frac{\lambda}{R} \ll 1, \quad \frac{\Delta_{\text{Au}}^{(s)} + \Delta_{\text{Al}}^{(s)} + \Delta^{(s)}}{R} \ll 1, \quad (8)$$

which are satisfied in our case with a wide safety margin. Using Eq. (7), one returns back to Eqs. (4) and (5) where the function $\Phi(r, \lambda)$ with any argument r is replaced with R [45, 48]. Now we compare the strength of constraints obtained using the exact Yukawa force and the approximate one derived using the PFA. Thus, at λ equal to 1 and $3 \mu\text{m}$ (recall that at larger λ the accuracy of the PFA is lower) the exact constraints are given by $\alpha < 2.19 \times 10^{11}$ and $\alpha < 3.62 \times 10^{10}$, respectively. These should be compared with respective PFA results $\alpha < 2.17 \times 10^{11}$ and $\alpha < 3.51 \times 10^{10}$. The comparison shows that the use of the PFA results in only 0.9% and 3.0% relative errors at $\lambda = 1$ and $3 \mu\text{m}$, respectively.

III. CONSTRAINTS FROM THE DYNAMIC AFM EXPERIMENT WITH A MAGNETIC PLATE

The experiment described in the previous section was first used for a selection between two different theoretical approaches to the Casimir force. Then we have used the measure of agreement between the selected approach and the measurement data for obtaining constraints on non-Newtonian gravity of the Yukawa type. This procedure is in fact well justified because the difference between the two approaches to the Casimir force cannot be modeled by the Yukawa interaction with some λ and α . It would be interesting, however, to independently verify the constraints obtained in such a way by using the measurement data consistent with the Lifshitz theory of the Casimir force without additional conditions. A good opportunity for this is provided by the recent measurement of the gradient of the Casimir force between the hollow Au-coated glass sphere and Si plate coated with ferromagnetic metal Ni [41].

The experiment on measuring the gradient of the Casimir force between magnetic and nonmagnetic metals was performed by means of a dynamic AFM in the configuration of a sphere and a plate. The layer structure of a sphere was the same as in the experiment of Sec. II, but its external radius was equal to $R = 64.1 \mu\text{m}$ [41]. The Si plate of density $\rho_{\text{Si}} = 2.33 \times 10^3 \text{ kg/m}^3$ was coated with a Ni layer of thickness $\Delta_{\text{Ni}}^{(p)} = 154 \text{ nm}$ and density $\rho_{\text{Ni}} = 8.9 \times 10^3 \text{ kg/m}^3$ with no additional intermediate layer. As a result, the exact expression for the gradient of the Yukawa force is again given by Eq. (4), where $X^{(s)}(\lambda)$ is contained in Eq. (5) and $X^{(p)}(\lambda)$ takes a more simple form

$$X^{(p)}(\lambda) = \rho_{\text{Ni}} - (\rho_{\text{Ni}} - \rho_{\text{Si}})e^{-\Delta_{\text{Ni}}^{(p)}/\lambda}. \quad (9)$$

The measurement data for the gradient of the Casimir force over the entire separation region from 220 to 500 nm were found consistent with the Lifshitz theory combined with the dielectric permittivities of Au and Ni. The latter were obtained by using the available optical properties of both metals extrapolated to zero frequencies by means of either the Drude or the plasma models. The important characteristic feature of this case is that, by coincidence, over the region of experimental separations the predictions of the Drude model approach almost coincide with the predictions of the plasma model approach. Thus, the experimental data for the gradient of the Casimir force were found in agreement with the

Lifshitz theory with no additional selection process among the theoretical models. This makes possible to use the magnetic experiment as an additional independent test for the constraints on corrections to Newtonian gravitation obtained previously.

The strongest constraints follow at the shortest separation distance $a = 220$ nm. They are obtained from Eq. (6), where now $\Delta_{F'}(a) = 0.79 \mu\text{N/m}$ (the larger experimental error determined at the same 67% confidence level as in the experiment of Sec. II is connected with the larger value of the sphere radius), by the substitution of Eq. (4), the quantity $X^{(s)}$ defined in Eq. (5) and $X^{(p)}$ defined in Eq. (9). The results are shown with line 2 in Fig. 1. As can be seen in Fig. 1, both the lines 1 and 2 show qualitatively similar constraints. In the region from $\lambda = \lambda_{\min} = 20$ nm to $\lambda = \lambda_0 = 0.5 \mu\text{m}$ the constraints of line 2 (the experiment with magnetic metal) are slightly stronger than the constraints of line 1 obtained from the experiment with nonmagnetic metals. Furthermore, in the separation region from $\lambda = \lambda_0 = 0.5 \mu\text{m}$ to $\lambda = \lambda_{\max} = 3 \mu\text{m}$ the constraints of the line 2 are slightly weaker than the constraints of the line 1. This is explained by different minimum separations, where these experiments have been performed, different sphere radii, and different materials of the plate used. There are, however, no qualitative differences that might be connected with the fact that the experimental data of the measurement with nonmagnetic bodies were used for making a selection between the two different theoretical approaches to the Casimir force, whereas the experimental data of the measurement with a magnetic plate were not. From line 2, at $\lambda = 20$ nm (the axion mass $m_a = 9.9$ eV) the interaction strength of the Yukawa-type correction to Newtonian gravity is constrained by $\alpha < 4.2 \times 10^{18}$ and at $\lambda = 3 \mu\text{m}$ (the axion mass $m_a = 66$ meV) by $\alpha < 5.6 \times 10^{10}$.

For comparison purposes in Fig. 2 we again plot line 2 representing the constraints on the Yukawa interaction obtained from the experiment using a magnetic plate [41] independently of any selection between the different theoretical approaches to the Casimir force. In the same figure, the constraints following from the dynamic determination of the Casimir pressure by means of a micromachined oscillator [19, 20] are shown by line 3, from the Casimir-less experiment [49], where the Casimir force was compensated, are shown by line 4, from measurement of the Casimir-Polder force between rubidium atoms belonging to the Bose-Einstein condensate and SiO_2 plate [35] are shown by line 5 [21], and from measurement of the Casimir force between smooth Au-coated sphere and Si plate covered with nanoscale trapezoidal corrugations [50] are shown by line 6 [51]. The constraints of Ref. [52] obtained

from an experiment [26] are not shown in Fig. 2 because they are stronger than those of lines 4 and 5 only at large interaction ranges $\log_{10}[\lambda(\text{m})] > -6.38$ and are not characterized by some definite confidence level. Note that the constraints of lines 3 and 4 were determined at a 95% confidence level, and the constraints of lines 5 and 6, as well as of line 2, were found at a 67% confidence level. From Fig. 2 it can be seen that although the constraints of line 2 are not the strongest ones, they are quite competitive within some interaction range and, as it is independent of a selection process between different theoretical approaches to the Casimir force, they provide additional support to the constraints obtained from other experiments.

IV. CONCLUSIONS AND DISCUSSION

In the foregoing, we have obtained constraints on the Yukawa-type corrections to Newton's gravitational law and light elementary particles which follow from the two recently performed experiments on the Casimir force. In the first of these experiments the gradient of the Casimir force between two Au surfaces has been measured by means of a dynamic AFM [24, 25]. This experiment, as well as the previous measurement performed using another laboratory technique [19, 20], was used for both the selection between two competing theoretical approaches to the Casimir force and for constraining corrections to Newtonian gravity. In the second recently performed experiment the dynamic AFM was used to measure the gradient of the Casimir force between a nonmagnetic metal Au and a ferromagnetic metal Ni [41]. The unique feature of this experiment is that in the region of experimental separations both competing approaches to the theoretical description of the Casimir force lead to nearly coincident results. For this experiment the unambiguous theoretical prediction was found in agreement with the experimental data and this fact has been used for obtaining constraints on the corrections to Newtonian gravity. Good agreement between the constraints obtained from both recent experiments and from other experiments on measuring the Casimir force performed earlier was demonstrated. This allows to conclude that in spite of the widely known problems in theory-experiment comparison discussed in the literature, measurements of the Casimir force in laboratory remain a reliable source of constraints on non-Newtonian gravity of the Yukawa-type and light elementary particles within the interaction range from nanometers to micrometers.

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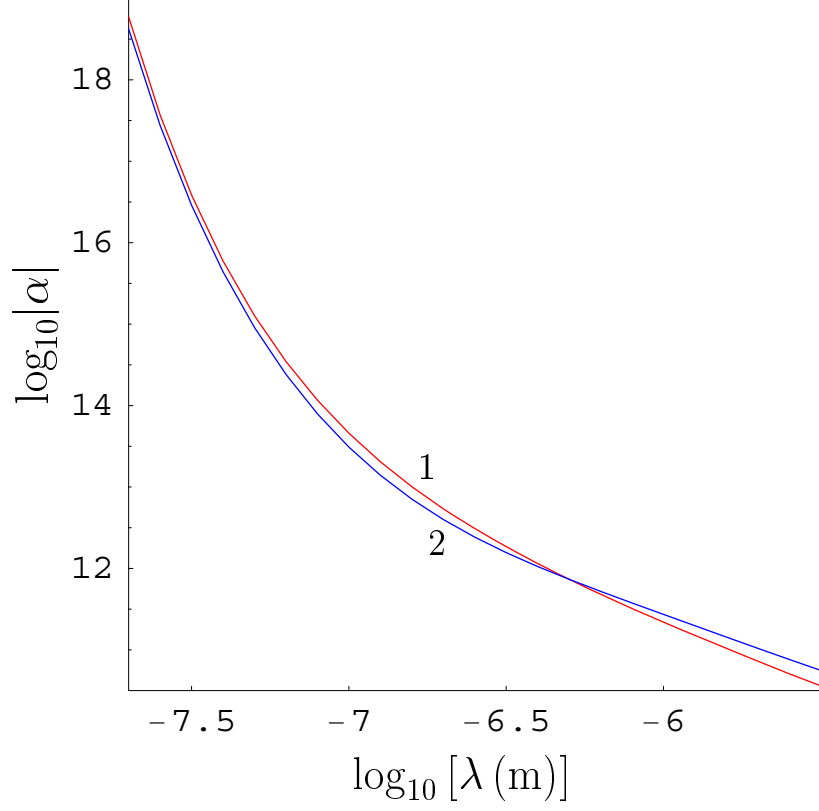


FIG. 1: (color online). Constraints on the parameters of Yukawa-type correction to Newton's gravitational law obtained from measurements of the gradient of the Casimir force by means of a dynamic AFM in the configuration of an Au-coated sphere and an Au-coated plate (line 1) and in the configuration of an Au-coated sphere and a magnetic Ni-coated plate (line 2). The regions of (λ, α) plane below each line are allowed and above each line are prohibited.

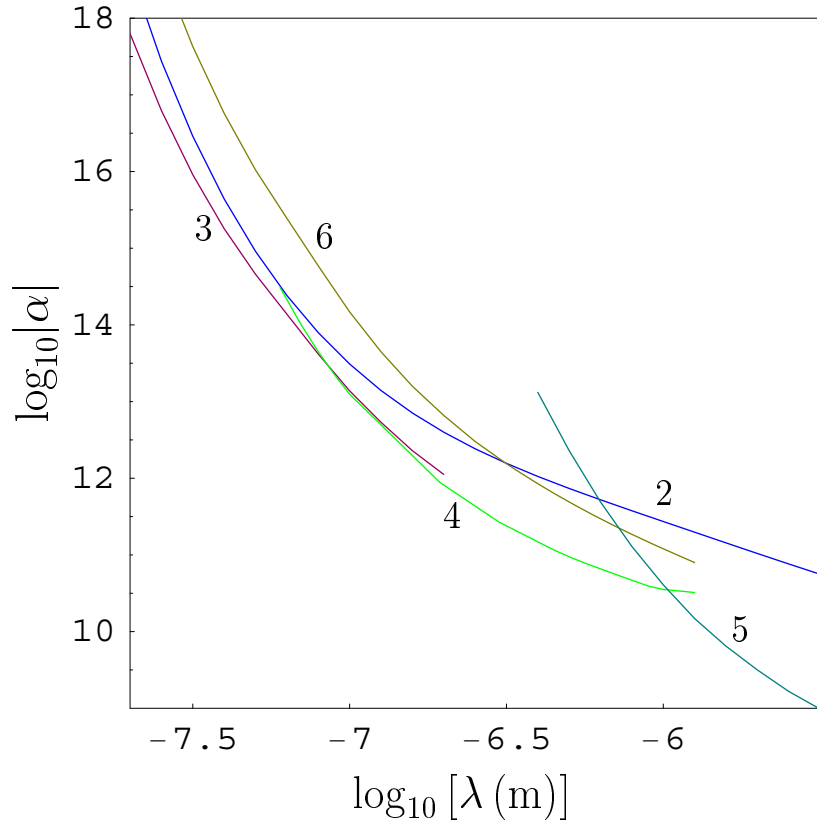


FIG. 2: (color online). Constraints on the parameters of Yukawa-type correction to Newton's gravitational law obtained from experiments on the Casimir force with a magnetic plate performed by means of a dynamic AFM [41] (line 2) and with two Au bodies performed by means of micromachined oscillator [19, 20] (line 3), from the Casimir-less experiment [49] (line 4), from measurements [35] of the Casimir-Polder force [21] (line 5), and from measurements of the Casimir force between an Au sphere and corrugated Si plate [50, 51] (line 6). The regions of (λ, α) plane below each line are allowed and above each line are prohibited.