1. (10 points) Let $X$ be a compact topological space and $A$ a subset of $X$.

(a) Define what is an accumulation point (or limit point) of $A$ in $X$.

(b) Show that if $A$ is an infinite subset of $X$ then $A$ must have at least one accumulation point in $X$.

2. (10 points) Show that $\mathbb{R}^n \setminus \{0\}$ is homeomorphic to $S^{n-1} \times \mathbb{R}$.

(Here $S^m$ is the unit sphere in $\mathbb{R}^{m+1}$).

3. (10 points) Let $X$ be a compact Hausdorff space and $f : X \to Y$ a quotient map.

Show that the following three statements are equivalent:

(a) $f$ is a closed map (i.e. it takes closed sets to closed sets.)

(b) $Y$ is a Hausdorff space.

(c) The set $\Gamma_f = \{(x_1, x_2) \in X \times X \mid f(x_1) = f(x_2)\}$ is closed in $X \times X$.

4. (10 points) Consider the subspace $Q = \bigcup_{i \geq 0} Q_i$ of $\mathbb{R}^2$, where $Q_0 = \{(0, y) \mid 0 < y \leq 1\}$ and $Q_i = \{(\frac{1}{i+1}, y) \mid 0 < y \leq 1\}$. Show that $Q$ is connected but not locally connected.

5. (10 points) On a square white piece of paper write in black ink the word TOPOLOGY and denote by $V$ the subspace consisting of the remaining white part.

(a) How many connected components does $V$ have?

(b) For every connected component of $V$ pick a point $x_0$ and compute $\pi_1(V, x_0)$. Justify all possible answers.

6. (10 points) Let $X$ and $\tilde{X}$ be two spaces that are path connected, locally path connected and such that $\pi_1(X) = \mathbb{Z}/23\mathbb{Z}$, and consider a covering map $p : \tilde{X} \to X$. Show that if $\tilde{X}$ is not the universal cover, then $p$ is a homeomorphism.

7. (10 points) Consider the bouquet of circles $B = \bigvee_{i=1}^{i=15} S^1$ as a canonical subset of the torus $T^{15} = \prod_{i=1}^{i=15} S^1$. Can $B$ be a retract of $T^{15}$? Justify your answer.