1. Let $Y = \{ z \in \mathbb{C} : z^3 \in [0, 1]\}$ (a triod) be a subspace of the complex plane $\mathbb{C}$, with the metric inherited from $\mathbb{C}$. Find 5 pairwise non-homeomorphic open balls in $Y$. Explain why they are not homeomorphic.

2. Prove that $\mathbb{R} \times S^1$ is homeomorphic with $\mathbb{R}^2 \setminus \{(0, 0)\}$.

3. In a metric space $(X, d)$ for nonempty sets $A, B \subset X$ define

$$\text{dist}(A, B) = \inf \{d(a, b) : a \in A, \ b \in B\}.$$ 

Prove that if $A$ is closed, $B$ compact and $A \cap B = \emptyset$ then $\text{dist}(A, B) > 0$.

4. Let $X$ be a compact space, $U$ its open subset, and $f : U \to [0, 1]$ a continuous map. Prove that the set

$$\{(x, t) : x \in U, 0 \leq t \leq f(x)\} \cup (X \setminus U) \times [0, 1] \subset X \times [0, 1]$$

is compact.

5. Let $A, B$ be subsets of $[0, 1]$. Let $X \subset \mathbb{R}^2$ be the union of closed segments joining the point $(0, 1)$ with all points of $A \times \{0\}$ and closed segments joining the point $(0, -1)$ with all points of $B \times \{0\}$. Prove that $X$ is connected if and only if $\overline{A} \cap B \neq \emptyset$ or $A \cap \overline{B} \neq \emptyset$.

6. Let $X$ be a topological space and let $S^2 \subset \mathbb{R}^3$ be the unit 2-sphere with the metric $d$ inherited from $\mathbb{R}^3$. Show that if $f, g : X \to S^2$ are continuous maps such that $d(f(x), g(x)) < 2$ for all $x \in X$ then $f$ and $g$ are homotopic.