1. Find the number of zeros of the polynomial

\[ P(z) = z^{2011} - 1000z^{1000} - 2011 \]

in the annulus \( \{ z \in \mathbb{C} : 1 < |z| < 2 \} \).

2. Evaluate the integral

\[ \int_{-\infty}^{\infty} \frac{dx}{x^2 + 14x + 113}. \]

Explain all steps.

3. Let \( D = \{ z : |z| < 1 \} \) be the unit disk. Does there exist an analytic function \( f : D \to \mathbb{C} \) such that

\[ f \left( \frac{1}{n} \right) = f \left( -\frac{1}{n} \right) = \frac{1}{n^3} \]

for \( n = 2, 3, 4, \ldots \)? Give an example of such a function or prove that it does not exist.

4. Find a conformal map from the set

\[ \{ z \in \mathbb{C} : |z| > 1 \} \setminus (-\infty, -1) \]

onto the set

\[ \mathbb{C} \setminus (-\infty, 0]. \]

You may represent your map as a finite composition of maps.

5. Find all singular points of the function

\[ f(z) = \frac{\sin(z^3)}{z^{21}} \]

and classify them.

6. Find all harmonic functions \( f : \mathbb{C} \setminus \{0\} \to \mathbb{R} \) that are constant on every circle centered at 0.