1. Describe and sketch the set \( \text{Im} \left( \frac{(z - 1)^3}{(z + 1)^3} \right) > 0 \).

2. Find all analytic functions \( f(z) \) defined in the unit disk \( \{ z \in \mathbb{C} : |z| < 1 \} \) such that \( \text{Im} f(z) > 0 \) for all \( z \), \( f(0) = i \) and \( f(1/3) = 2i \).

3. Let \( f(z) \) be an analytic function in the disk \( \{ z \in \mathbb{C} : |z| < R \} \), where \( R > 1 \).

Prove that \( \int_{|z|=1} f(z) \, dz = 2\pi i \frac{f'(0)}{1} \). The integral is for the positive sense of the circle.

4. Let \( f(z) = \frac{\log(z^2 + 1)}{(z + 2)^2} \), where \( -\pi \leq \text{Im} \log(z^2 + 1) < \pi \). Write the Laurent series for \( f(z) \) around \( z = -2 \). Determine if the obtained series is convergent at \( z = 1 \), \( z = 0 \), \( z = -1 \), and find the sum of the series at the points where the series is convergent.

5. Define by induction: \( a_0 = 1/2 \), \( a_1 = 1/3 \), and \( a_n = a_{n-1} a_{n-2} \) for \( n \geq 2 \). Prove that the series \( f(z) = \sum_{n=0}^{\infty} a_n z^n \) is convergent for all \( z \). Show that there exists a sequence \( z_1, z_2, \ldots \) such that \( \lim_{n \to \infty} z_n = \infty \) and \( \lim_{n \to \infty} f(z_n) = 0 \).

6. Evaluate \( \int_0^\infty \frac{x \sqrt{x}}{x^3 + 1} \, dx \) using residues. Simplify the answer as much as possible.