(1) Let $X = [0, 3]/(1, 2)$ be the quotient space of the interval $[0, 3]$ in which all points of the interval $(1, 2)$ are identified. Prove that

(a) $X$ is connected.

(b) $X$ is compact.

(c) $X$ is not Hausdorff.

(2) Which one of the following are true? If true, prove it. If false, give a counterexample.

(a) If $A$ is connected, then $\overline{A}$ is connected.

(b) If $A$ is path-connected, then $\overline{A}$ is path-connected.

(3) Show that a space $X$ is simply connected if and only if any two continuous maps $f, g : S^1 \to X$ are homotopic to each other.

(4) Consider two different points $x_0, x_1$ in the sphere $S^2$. Compute the fundamental group of the quotient space $S^2/x_0 \sim x_1$, where the two points are identified.

(5) Consider a connected and locally path-connected space $X$ such that $\pi_1(X)$ is finite. Show that every continuous map $f : X \to S^1$ is homotopic to the constant map.

(6) Prove that there is no retraction from the sphere $S^2$ onto its equator $E$.

(7) Let $f : X \to Y$ be a covering map.

(a) Prove that $f$ is open.

(b) Prove that if $Y$ is compact and $f$ is an $n$-fold covering with finite $n$, then $X$ is compact.