1. Find a one-to-one conformal mapping from the upper half-plane without the segment $(0, i]$ onto the unit disk (the answer can be given as a series of maps).

2. (a) Determine the number of points $z$ in the unit disk such that $e^z = 3z^4$.
    (b) Determine the number of points $z$ in the complex plane such that $e^z = 3z^4$.

3. Use contour integration to compute the integral
   \[ \int_{-\infty}^{\infty} \frac{dx}{x^2 + 2x + 2}. \]

4. Suppose $f(z)$ is an entire function such that $|f(z)| \leq A |z|$ for all $z$, where $A$ is some fixed positive number. Show that $f(z) = az$, where $a$ is a complex constant.

5. Compute the Laurent series representation of $\frac{e^z}{(z+1)^2}$ around $z = -1$ and show that it is valid for $0 < |z+1| < \infty$.

6. Let $\gamma$ be the circle $|z| = 2$ described counterclockwise. Evaluate the integral
   \[ \int_{\gamma} \frac{5z - 2}{z(z - 1)} \, dz \]
   (a) by finding the residues at the finite poles,
   (b) by finding the residue at $\infty$.

7. Compute
   \[ \frac{1}{2\pi i} \int_{\gamma} \frac{z^3 + z}{z - z_0} \, dz, \]
   where $z_0$ stands for each of $-1, 0, 1, i$ and $\gamma$ is the curve below

8. Let $P(z) = \sum_{k=0}^{n} a_k z^k$ be a polynomial of degree $n$. Prove that if $|a_0| < |a_n|$, then $P$ has at least one zero inside the unit disk.