1. (15) Find the general solution of the first order linear PDE:

\[ u_x + u_y + 2u_z + u + x^2 = 0 \]

2. (15) Separating variables in rectangular coordinate system, solve

\[ \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b, \]

\[ u|_{x=0} = A \sin \frac{\pi y}{b}, \quad u|_{x=a} = 0 \]

\[ u|_{y=0} = B \sin \frac{\pi x}{a}, \quad u|_{y=b} = 0 \]

3. (15) Evaluate the integral

\[ \int_0^{2\pi} \frac{\sin(n\phi)}{a^2 + r^2 - 2ar \sin \phi} d\phi, \]

where \( n \) is an integer and \( 0 < r < a \). (Hint: Use the Poisson integral formulae for the solution of the Dirichlet problem for a disc.)

4. (15) Let \( \Omega \) be a bounded normal domain in \( \mathbb{R}^n \) and let \( u(x) \) be a solution of the Dirichlet problem

\[ \Delta u = -1, \quad x \in \Omega, \]

\[ u = 0, \quad x \in \partial \Omega. \]

where does \( u(x) \) attain its minimum ?

5. (10) Let \( \Omega \) be a bounded normal domain in \( \mathbb{R}^n \). Prove that the boundary problem,

\[ \Delta u = f, \quad x \in \Omega, \]

\[ \frac{\partial u}{\partial n} = 1, \quad x \in \partial \Omega, \]

has no solution if \( f \in C(\partial \Omega) \) and \( f < 0 \).
6. (15) Suppose that $D \subset \mathbb{R}^m$ is a bounded open set, and consider the unbounded open set $\Omega = \mathbb{R}^m \setminus \overline{D}$. Suppose that $u \in C^2(\Omega) \cap C(\overline{\Omega})$ is a harmonic function in $\Omega$ such that

$$u = 0 \quad \text{on} \quad \partial \Omega \quad \text{and} \quad \lim_{|x| \to \infty} u(x) = 0. \quad (1)$$

prove that it must be $u \equiv 0$ on $\Omega$ (Maximum Principle for unbounded domains).

7. (15) Let $\Omega$ be a bounded normal domain in $\mathbb{R}^n$. Show that if $u$ satisfies the hyperbolic equation,

$$\Delta u = u_{tt} + \gamma u_t, \quad x \in \Omega, \quad t > 0 \quad (2)$$

and the Dirichlet boundary condition,

$$u = 0, \quad x \in \partial \Omega, \quad t \geq 0, \quad (3)$$

where $\gamma = \gamma(x, t)$ is a nonnegative continuous function defined for $x \in \overline{\Omega}, \ t \geq 0$, then

$$\int_{\Omega} (|\nabla u|^2 + u_t^2)|_{t=T} \, dx \leq \int_{\Omega} (|\nabla u|^2 + u_t^2)|_{t=0} \, dx.$$ 

Using this inequality, prove the uniqueness of the solution of the initial-boundary problem associated with (2), (3).